(6 pages) **Reg. No. :** ....

Code No. : 40353 E **Sub. Code : JSMA 4 A/ JSMC 4 A**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fourth Semester

Mathematics/Mathematics with CA — Main

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORMS AND FOURIER SERIES

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $\cos 4\theta + 4\cos 2\theta + 3 =$ ____________

   (a) $2^3 \cos^4 \theta$ (b) $\cos^4 \theta$

   (c) $2^4 \cos^4 \theta$ (d) $2\cos^4 \theta$

2. $16\sin^4 \theta - 20\sin^2 \theta + 5 =$ ____________

   (a) $\sin 5\theta$ (b) $\dfrac{\sin 5\theta}{\sin \theta}$

   (c) $\sin 4\theta$ (d) $\dfrac{\sin 4\theta}{\sin \theta}$

3. $\cos(ix) =$ ____________

(a) $\cos x$ (b) $i\cos x$

(c) $\cos h\, x$ (d) $i\cos h\, x$

4. $Log(-1) =$ ____________

(a) $i2n\pi$ (b) $-i2n\pi$

(c) 0 (d) $i(2n+1)\pi$

5. $L(t \cos t) =$ ____________

(a) $\frac{1}{s^2+1}$ (b) $\frac{s^2}{s^2+1}$

(c) $\frac{s^2-1}{s^2+1}$ (d) $\frac{s^2-1}{(s^2+1)^2}$

6. $L^{-1}\left(\frac{1}{s^2}\right) =$ ____________

(a) 1 (b) $\frac{1}{t}$

(c) t (d) t^2

7. If $y(0) = y'(0) = 0$, then $L(y'') =$

(a) 0 (b) 1

(c) $s^2 L(y)$ (d) $s L(y)$

8. $L^{-1}\left(\frac{1}{s-2}\right)=$

(a) $t-2$ (b) e^{2t}

(c) e^{-2t} (d) $2e^{t}$

9. If $f(x)$ is an even function, then

(a) $f(x)=f(-x)$ (b) $f(x)=-f(-x)$

(c) $f(x)=f(x^2)$ (d) $f(x)=f(f(x))$

10. If $f(x)$ is an odd function defined in $(-l, l)$, then in the Fourier expansion $a_0 =$ ______________

(a) 0 (b) l

(c) $2l$ (d) $\frac{\pi}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) Expand $\cos 5\theta$ interms of powers of $\cos\theta$.

Or

(b) Prove: $2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta +$

$15\cos\theta + 10$.

12. (a) Prove: $\cos h^{-1} x = \log_e\left(x+\sqrt{x^2-1}\right)$.

Or

(b) Seperate into real and imaginary parts: $\tan^{-1}(x+iy)$.

13. (a) Find: $L\left(\frac{1-e^t}{t}\right)$.

Or

(b) Find: $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$.

14. (a) Solve: $\frac{d^2 y}{dt^2}+2\frac{dy}{dt}+5y=4e^{-t}$ given that $y=0, \frac{dy}{dt}=0$ when $t=0$.

Or

(b) Solve: $\frac{d^2y}{dt^2}+4y=A\sin kt$ given that $y=\frac{dy}{dt}=0$ when $t=0$.

15. (a) Find the Fourier sine series of $f(x)=x$ in $0<x<2$.

Or

(b) Find the Fourier expansion of the function

$$f(x)=\begin{cases}\pi+2x, & -\pi<x<0\\ \pi-2x, & 0\leq x\leq \pi\end{cases}$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 600 words.

16. (a) Prove : $\dfrac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

Or

(b) Expand $\cos^5\theta\sin^3\theta$ in a series of sires of multiples of θ.

17. (a) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$, then show that $\cos 2\theta \cos h\, 2\phi = 3$.

Or

(b) Prove : $\dfrac{\sin\theta}{1!} + \dfrac{\sin 2\theta}{2\,!} + \ldots = e^{\cos\theta}\sin(\sin\theta)$.

18. (a) Prove:

(i) $L(tf(t)) = \dfrac{-d}{ds}F(s)$

(ii) $L\left(\dfrac{f(t)}{t}\right) = \int_s^\infty f(s)\,ds$.

Or

(b) Find

(i) $L^{-1}\left(\dfrac{s+2}{\left(s^2+4s+5\right)^2}\right)$

(ii) $L^{-1}\left(\dfrac{s}{\left(s+3\right)^2+4}\right)$.

19. (a) Solve: $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}-3y=\sin t$; given that $y=\dfrac{dy}{dt}=0$ when $t=0$.

Or

(b) Solve : $\dfrac{dx}{dt}-\dfrac{dy}{dt}-2x+2y=1-2t$;

$\dfrac{d^2x}{dt^2}+2\dfrac{dy}{dt}+x=0$; given that

$x=0,\ y=0,\ \dfrac{dx}{dt}=0$ when $t=0$.

20. (a) Find the Fourier series of $f(x)=x^2$ in $(-\pi, \pi)$.

Or

(b) Find the Fourier cosine series of $f(x)=\pi-x$ in $(0, \pi)$.
