(6 pages) **Reg. No. :** ....

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M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which of the following is the Non-Homogeneous Equation?

   (a) $y''=0$ (b) $y'=y$

   (c) $y''=y$ (d) $y'=e^x$

2. Any linear combination of two solutions of the homogeneous equation $y''+P(x)y'+Q(x)y=0$ is also a ————

   (a) solution (b) equation

   (c) IVP (d) BVP

3. Which of the following is the Transcendental Equation?

   (a) $x=0$ (b) $y=0$

   (c) $z=0$ (d) $e^x=0$

4. Any point that is not ordinary point of the equation $y''+P(x)y'+Q(x)y=0$ is called

   (a) Singular point

   (b) Special function piont

   (c) Ordinary point

   (d) Point function

5. $P_n = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ is called ———— formula.

   (a) Legendre (b) Rodrigues

   (c) Binomial (d) Bessel

6. $y = a_0x^m + a_1x^{m+1} + ...$ is called ———— series.

   (a) Frobenius (b) Rodrigues

   (c) Binomial (d) Bessel

7. $\Gamma(6) =$ ————

   (a) 20 (b) 120

   (c) 100 (d) 40

8. $\Gamma\left(\frac{5}{2}\right) =$ ——————

(a) 1.32 (b) 2.32

(c) 0 (d) ∞

9. If W(t) is the Wronskian of the two solutions of the homogeneous system then W(t) is —————— on [a,b].

(a) Identically Zero (b) Never zero

(c) either (a) or (b) (d) zero

10. The system $\frac{dx}{dt} = a(t)x + f(t), f(t) = 0$ then this system is called ——————

(a) Homogeneous (b) non-Homogeneous

(c) non-linear (d) Wronskian

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve $y''+y'=0$.

Or

(b) Find the particular solution for $y''+y=\csc x$.

12. (a) Prove that the equation $(t-2)x''+x=0$ does not has an ordinary point t = 2.

Or

(b) Find the general solution for $y''+y=0$.

13. (a) Determine the nature of Singularity of $f(z)=\frac{e^z}{z}$.

Or

(b) Discuss the nature of Singularity of $f(z)=\frac{1}{\sin(\cos z)}$.

14. (a) Prove that $\frac{d}{dt}\left[t^{-p}T_{p+1}(t)\right] = -\left[t^{-p}T_{p+1}(t)\right]$

Or

(b) Find the general solution of the equation

$$9x^2y''+9xy'+\left(9x^2-\frac{1}{4}\right)y=0.$$

15. (a) Find the Wronkian value W of the equation

$$\frac{d^2y}{dx^2}+4y=\tan 2x$$

Or

(b) Find the Complementary function for the differential equation.

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x.$$

SECTION C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $y_1(x)$ and $y_2(x)$ are two solutions of $y''+P(x)y'+Q(x)y=0$ on [a,b], then prove that their Wronskian $W=W(y_1,y_2)$ is either identically zero or never zero on [a,b].

Or

(b) Show that $y=C_1\sin x+C_2\cos x$ is the general solution of $y''+y'=0$ on any interval, and find the particular solution for which $y(0)=Z \ and \ y'(0)=3$.

17. (a) Find the power series solution for the equation $y'=t^2-y^2, y(0)=0$ for $t=0$.

Or

(b) Find the power series solution for the equation $(1+x)y'=Py, y(0)=1$.

18. (a) Consider the equation $t(t-1)^2(t+3)x''+t^2x'-(t^2+t-1)x=0$. Check whether the point t = 1 is the regular Singular point or not.

Or

(b) If P_n is the Legendre polynomial, the prove that $\int_{-1}^{1} P_n^2(t)dt=\frac{2}{2n+1}$.

19. (a) Consider the differential equation $4x^2y''+4xy'+\left(x-\frac{1}{36}\right)y=0$. Set $Z=\sqrt{x}$ and reduce the differential equation to a Bessel equation in $Z, \frac{dy}{dz} \ and \frac{d^2y}{dz^2}$.

Or

(b) Prove that $P_n'(1)=\frac{1}{2}n(n+1)$.

20. (a) Find the general solution of $\frac{dx}{dt}=x+y; \frac{dy}{dt}=4x-2y$.

Or

(b) Find the general solution of $\frac{dx}{dt}=3x-4y; \frac{dy}{dt}=x-y$.