(8 pages) **Reg. No. :** ....

Code No. : 7835 **Sub. Code : PMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics – Core

NUMERICAL ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $f(x)=\frac{1}{x^2}$ whose arguments are a, b, c then the first divided differences is ————.

(a) $\frac{-(a+b)}{a^2b^2}$ (b) $\frac{ab+bc+ca}{a^2b^2c^2}$

(c) $\frac{-1}{abcd}$ (d) $\frac{ab+bc}{a^2b^2c^2}$

2. Striling's formula give the most accurate result for the values of P lying between ————.

(a) $\frac{1}{4}\le P\le\frac{3}{4}$ (b) $0<P<1$

(c) $\frac{-1}{4}\le P\le\frac{1}{4}$ (d) $-1<P<0$

3. $\left(\frac{dy}{dx}\right)_{x=x_n}$ = ————.

(a) $\frac{1}{h}\left[\nabla y_n+\nabla^2 y_n+\nabla^3 y_n+.....\right]$

(b) $\frac{1}{h}\left[\nabla^2 y_n+\nabla^3 y_n+\frac{1}{3}\nabla^4 y_n+.....\right]$

(c) $\frac{1}{h}\left[\nabla y_n+\frac{1}{2}\nabla^2 y_n+\frac{1}{3}\nabla^3 y_n+.....\right]$

(d) $\frac{1}{h}\left[\nabla y_n+\frac{1}{2!}\nabla^2 y_n+\frac{1}{3!}\nabla^3 y_n+.....\right]$

4. $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$ = ————.

(a) $\frac{1}{h}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0-......\right]$

(b) $\frac{1}{h^2}\left[\frac{\Delta^2 y_0}{2!}-\frac{\Delta^3 y_0}{3!}+\frac{11}{12}\Delta^4 y_0-......\right]$

(c) $\frac{1}{h^2}\left[\Delta^2 y_0+\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+......\right]$

(d) $\frac{1}{h^2}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+......\right]$

5. The value of $\int_0^6 \frac{dx}{1+x}$ using Weddley's rule is ______.

(a) 1.95285 (b) 2.95285

(c) 0.95285 (d) 1.75285

6. If $I_1 = 0.775$ and $I_2 = 0.7828$ then by using Romberg's formula for I_1 and $I_2 =$ ______.

(a) 0.7854 (b) 0.7825

(c) 0.6825 (d) 0.7835

7. $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ then $y(0.1)$ is ______.

(a) 0.1 (b) 0.2

(c) 1 (d) 0

8. ______ method is the Runge – Kutta method of second order.

(a) Euler's method (b) Picard's method

(c) Modified Euler's (d) Taylor's method

9. Adam's corrector formula is ______.

(a) $y_{n+1}, C = y_n + \frac{h}{2!}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} + y'_{n-2}\right]$

(b) $y_{n+1}, C = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} - y'_{n-2}\right]$

(c) $y_{n+1}, C = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$

(d) $y_{n+1}, C = y_n + \frac{h}{2!}\left[9y'_{n+1} + 19y'_n - 5y'_{n+1} - y'_{n-2}\right]$

10. For solving differential equation in Milne's predictor formula the required number of initial values are ______.

(a) 4 (b) 3

(c) 2 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a cubic polynomial which takes the following values.

x	0	1	2	3
$f(x)$	1	2	1	10

Or

(b) Find the equation of the cubic curve which passes through the points (4, –43), (7, 83), (9, 327) and (12, 1053). Hence find $f(10)$.

12. (a) Find $\frac{dy}{dx}$ at $x = 51$ from the following data

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

Or

(b) Give $u_0 = 5$, $u_1 = 15, u_2 = 57$ and $\frac{du}{dx} = 4a + x = 0$ and 72 at $x = 2$ find $\Delta^3 u_0$ and $\Delta^4 u_0$.

13. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$. Hence determine the value of π.

Or

(b) Calculate $\int_{0.5}^{0.7} e^{-x} x^{\frac{1}{2}} dx$ taking 5 ordinates by Simpson's $\frac{1}{3}$ rule.

14. (a) Using Taylor's method, find $y(0.1)$ correct to 3 decimal places from $\frac{dy}{dx} + 2xy = 1$, $y_0 = 0$.

Or

(b) Using Picard's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$.

15. (a) Using Milne's predictor corrector method find $y(0.4)$ for the differential equation $\frac{dy}{dx} = 1 + xy, y(0) = 2$.

Or

(b) Using Adam's Bashforth method find $y(0.4)$ for the differential equation $y' = 1 + xy$, $y(0) = 2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Using Striling's formula compute y_{35} given that $y_{10} = 600$; $y_{20} = 512$; $y_{30} = 439$; $y_{40} = 346$; $y_{50} = 243$.

Or

(b) Tabulate $y = x^3$ for $x = 2,3,4,5$ and calculate the cube root of 10 correct to three decimal places.

17. (a) Find $y'(x)$ given

x	0	1	2	3	4
$y(x)$	1	1	15	40	85

Hence find $y'(x)$ at $x = 0.5$.

Or

(b) A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of time t (seconds).

t	0	0.2	0.4	0.6	0.8	1.0
θ	0	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.6$ seconds.

18. (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method correct to 4 decimal places. Hence deduce an approximate value of π.

Or

(b) Evaluate $I = \int_1^2\int_1^2 \left(\frac{1}{x+y}\right) dx\,dy$ using Trapezoidal rule with $h = K = 0.25$.

19. (a) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y(1) = 1$, Evaluate $y(1.3)$ by modified Euler's method.

Or

(b) Compute $y(0.1)$ and $y(0.2)$ by Runge–Kutta method of 4th order for the differential equation $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

20. (a) Given $\frac{dy}{dx} = \frac{1}{x+y}$; $y(0) = 2$. If $y(0.2) = 2.09$, $y(0.4) = 2.17$ and $y(0.6) = 2.24$ find $y(0.8)$ using Milne's method.

Or

(b) Using Adams – Bashforth method, determine $y(1.4)$ given that $y' - x^2y = x^2$, $y(1) = 1$. Obtain the starting values from Euler's method.