KAMARAJ COLLEGE (Autonomous)

Accredited with A+ Grade by NAAC

(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)
THOOTHUKUDI – 628 003

6 Pages **Reg. No: ……………………………………………….**

Question. Code No : 24O0141 **Sub Code : 24PMCA11**

PG Degree - End Semester Examinations, November 2024

First Semester

M.C.A

Major - DISCRETE MATHEMATICS

(For those who joined in July 2024 onwards)

Time : 3 Hours **Maximum : 75 Marks**

PART A – (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

1. The number of relations from A to B with $|A| = m$ and $|B| = n$ is ___

(a) mn (b) 2^n

(c) 2^m (d) 2^{mn}

2. If f, g, h are functions from R to R defined by $f(x) = x + 1$, $g(x) = x^2 + 2, h(x) = 2x + 1$, then $(h \circ g \circ f)(2) =$ ________

(a) 23 (b) 20
(c) 21 (d) 22

3. $\neg P \vee (Q \wedge \neg P)$ is equivalent to _____

(a) Q (b) P
(c) $\neg$ Q (d) $\neg$ P

4. A valid conclusion from $P \vee Q$ and $\neg P$ is ______

(a) $P \vee Q$ (b) $P \wedge Q$
(c) P (d) Q

5. The general solution of the relation $a_n = 4(a_{n-1} - a_{n-2})$ is ___

(a) $(A + Bn)2^n$ (b) $A2^n + B(-2)^n$
(c) $(A + B)2^n$ (d) $(A + B)(-2)^n$

6. If $a_{n+1} - da_n = 0, a_3 = 189, a_5 = 1705$, then d $=$ _____

(a) 3 (b) ± 3
(c) -3 (d) 3^2

7. The inverse of the matrix $\begin{bmatrix} 1 & 5 \\ -2 & -9 \end{bmatrix}$ is ____

(a) $\begin{bmatrix} -1 & -5 \\ 2 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 \\ 5 & 9 \end{bmatrix}$
(c) $\begin{bmatrix} -9 & -5 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 2 & 9 \end{bmatrix}$

8. If $\begin{bmatrix} x & 0 \\ 1 & y \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$, then x, y =-------

(a) 3, 2 (b) $-3, -2$
(c) $3, -2$ (d) $-3, 2$

9. A tree with 5 vertices can be ______

(a) D_5 (b) P_5
(c) C_5 (d) W_5

10. If G is a 2 - regular graph with 16 edges, then the number of vertices of G is _____
(a) 4 (b) 16
(c) 8 (d) 32

PART B – (5X5=25Marks)

Answer ALL Questions choosing either (a) or (b).

(Answer should not exceed 250 words.)

11. (a) Let R be a relation from A to B and S a relation from B to C. Then prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

(OR)

(b) If $f : A \to B$ and $g : B \to C$ are invertible, then prove that $g \circ f : A \to C$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

12. (a) Construct the truth table for the formula

$$\propto = (P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$$

(OR)

(b) Prove the Absorption Law using truth table.

13. (a) Find a recurrence relation for $a_n = A.2^n + B.(-3)^n$.

(OR)

(b) Solve the following recurrence relation: $a_n + a_{n-2} = 0, a_0 = 3, a_1 = 0$

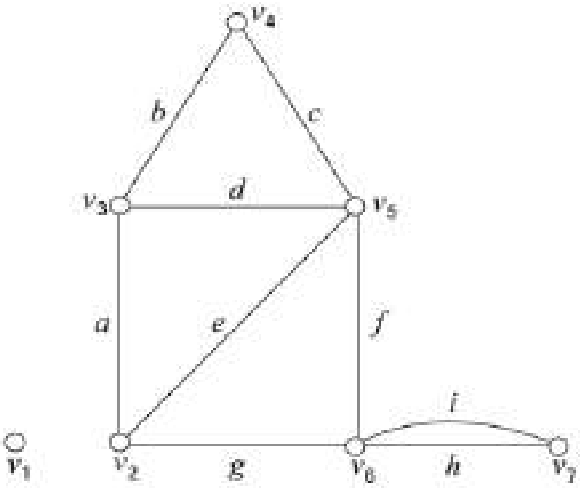
14. (a) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{pmatrix}$.

(OR)

(b) Using Cramer's Rule, solve the system of equation

$$12x + 3y = 15, 2x - 3y = 13$$

15. (a) Find the incidence martrix of the following graph:



(OR)

(b) Draw all trees with one, two, three, four and five vertices.

PART C - ($5 \times 8 = 40$ Marks)

Answer ALL Questions choosing either (a) or (b).

(Answer should not exceed 600 words.)

16. (a) Prove that the following relations are Equivalence Relations:

    i) $\{(1,1),(1,2),(2,1),(2,2),(3,3)\}\ in\ \{1,2,3\}$

    ii) $(a,b)R(c,d)\ if\ ad = bc\ where\ a,b,c,d \in Z^+$

(OR)

(b) Let $f: R \to R$ and $g: R \to R$ be defined by $f(x) = x^2 - 3$ and $g(x) = x + 4$. Find $g \circ f$ and $f \circ g$. Are they injective? Surjective?

17. (a) Using Quine's method, show that $(P \to R) \to ((Q \to R) \to (P \vee Q \to R))$ is tautology.

(OR)

(b) i) Obtain the principal disjunctive normal form of

$$\propto = (\neg P \vee \neg Q) \to (\neg P \wedge R)$$

ii) Find the principal conjunctive normal form of

$$\propto = P \vee (Q \to R).$$

18. (a) Solve the Fibonacci recurrence relation $F_n = F_{n-1} + F_{n-2}, F_1 = F_2 = 1.$

(OR)

(b) Solve the following recurrence relations:

i) $a_n - 7a_{n-1} + 10a_{n-2} = n.4^n$

ii) $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 0.$

19. (a) State and Prove Cayley Hamilton Theorem.

(OR)

(b) Find the inverse of the matrix $\begin{pmatrix} 4 & 2 & -3 \\ 1 & -1 & 2 \\ 5 & 3 & 0 \end{pmatrix}$

20. (a) In a complete graph K_n with n vertices, n being odd, prove that there are $(n-1)/2$ edge - disjoint Hamiltonian circuits.

(OR)

(b) Prove that K_5 is not planar.