KAMARAJ COLLEGE (Autonomous)

Accredited with A+ Grade by NAAC

(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

THOOTHUKUDI – 628 003

(6 Pages) **Reg. No: ……………………………………………**

Question Code No : 25O00807 **Sub Code : 24PMMA32**

PG Degree - End Semester Examinations, November 2025

Third Semester

M.Sc. MATHEMATICS

Complex Analysis

(For those who joined in July 2024 onwards)

Time : 3 Hours **Maximum : 75 Marks**

PART– A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer:

1. The Cauchy-Riemann equation of $f(z) = u(x) + iv(x)$ is ________________.

   (a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

   (c) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y} = -\frac{\partial v}{\partial x}$ (d) $\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y}, \frac{\partial v}{\partial y} = \frac{\partial v}{\partial x}$

2. The Hadamard's formula for the radius of convergence is

____

(a) $R = \limsup_{n\to\infty} \sqrt[n]{|a_n|}$ (b) $R = \liminf_{n\to\infty} \sqrt[n]{|a_n|}$

(c) $\frac{1}{R} = \limsup_{n\to\infty} \sqrt[n]{|a_n|}$ (d) $\frac{1}{R} = \liminf_{n\to\infty} \sqrt[n]{|a_n|}$

3. The value of $n(\gamma, 0) =$____________.

(a) 1 (b) 0

(c) ∞ (d) 2

4. A function $f(z)$ which is analytic in a region Ω expect for poles is said to be _______ in Ω.

(a) Meromorphic (b) Poles

(c) Homeomorphic (d) Singularities

5. The integral of an exact differential over any cycle is_______.

(a) Chain (b) One

(c) Connected (d) Zero

6. A cycle γ is an open set Ω is said to be homologous to zero with respect to Ω if ___________.

(a) $\Omega \sim 0 (mod\, \gamma)$ (b) $\gamma \sim 0 (mod\, \Omega)$

(c) $\Omega \sim 1 (mod\, \gamma)$ (d) $\gamma \sim 1 (mod\, \Omega)$

7. The value of the integral $\int_0^\infty \frac{\sin x}{x} dx =$____________.

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. If u_1 and u_2 are harmonic in a region Ω, then $\int_{\gamma} u_1 * du_2 - u_2 * du_1 =$ __________

(a) $u_2 u_1$ (b) u_2

(c) 0 (d) u_1

9. The value of $P_c =$ __________.

(a) 1 (b) 0

(c) c (d) P

10. If $R_1 = 0$, the point a is __________ singularity.

(a) Non isolated (b) Constant

(c) Zero (d) Isolated

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

11. (a) Prove that the function $u + iv$ determined by a pair of conjugate harmonic function is always analytic.

(OR)

(b) Find the radius of convergence of the series $\sum n^p z^n$.

12. (a) Examine a function which is analytic and bounded in the whole plane must reduce to a constant.

(OR)

(b) A non-constant analytic function maps open sets on to open sets - Justify.

13. (a) If $f(z)$ is analytic in a region Ω, then prove that $\int_{\gamma} f(z)dz = 0$ holds for every cycle γ in Ω.

(OR)

(b) If $f(z) \neq 0$ is analytic in a simply connected region Ω, then prove that it is possible to define single valued analytic branches of $\log f(z)$ and $\sqrt[n]{f(z)}$ in Ω.

14. (a) Evaluate $\int_0^{\pi} \frac{1}{\alpha + \cos\theta} d\theta, \alpha > 1$ by the methods of residues.

(OR)

(b) If $u(z)$ is harmonic for $|z| < R$ continuous for $|z| \leq R$ then prove that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z)\, d\theta$.

15. (a) State and prove the Weierstrass's theorem.

(OR)

(b) State and prove Schwarz theorem.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

16. (a) Explain Hadamard's formula for the radius of convergence.

(OR)

(b) State and prove the Lucas's theorem.

17. (a) Analyze a function of the index $n(\gamma, a)$ is constant in each of the regions determined by γ and zero in the unbounded region.

(OR)

(b) Verify that an analytic function comes arbitrary close to any complex value in every neighbourhood of an essential singularity.

18. (a) Demonstrate that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.

(OR)

(b) If f(z) is meromorphic in Ω with the zeros a_j and the poles b_k, then evaluate the value of the integral $\int_\gamma \frac{f'(z)}{f(z)} dz$, for every cycle γ which is homologous to zero in Ω.

19. (a) Evaluate $\int_0^{\pi} \log \sin \theta d\theta$ by the methods of residues.

(OR)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$, by using residues.

20. (a) Develop arc tan z in the power of z upto the term z^5.

(OR)

(b) Prove that every analytic function can be developed in a convergent Taylor series.