KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:...**

Question Code: 26E00806 **Course Code : 24PMMA32**

PG Degree - End Semester Examinations, April 2026

Third Semester

M.Sc., MATHEMATICS

Complex Analysis

(For those who joined in July 2024 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A ($10 \times 1 = 10$ Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:2 1. An analytic function is independent of

(a) z (b) $\bar{z}$

(c) $|z|$ (d) constant term

CO:1 K:2 2. The radius of convergence of $\sum \frac{z^n}{n!}$ is

(a) 0 (b) 1

(c) n (d) ∞

CO:2 K:1 3. $n(C, z) =$ _____ for all points z inside of a circle C about a in Δ.

(a) 0 (b) 1

(c) n (d) ∞

CO:2 K:1 4. The point a is a pole of $f(z)$ if $\lim_{z \to a} f(z) =$

(a) ∞ (b) $f(a)$

(c) $f'(a)$ (d) 0

CO:3 K:1 5. A chain is _____ if it can be represented as a sum of closed curves.

(a) a circle (b) a closed chain

(c) a cycle (d) connected

CO:3 K:2 6. The residue of $\frac{e^z}{(z-a)^2}$ at $z = a$ is

(a) 0 (b) e

(c) e^a (d) ∞

CO:4 K:2 7. $\int_0^\infty \frac{\sin x\, dx}{x} =$

(a) π (b) $\pi/2$

(c) 2π (d) ∞

CO:4 K:1 8. The conjugate differential of du is

(a) $\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$ (b) $\frac{\partial u}{\partial x} dx - \frac{\partial u}{\partial y} dy$

(c) $\frac{\partial u}{\partial x} dy - \frac{\partial u}{\partial y} dx$ (d) $\frac{\partial u}{\partial x} dy + \frac{\partial u}{\partial y} dx$

CO:5 K:2 9. In the Poisson integral, $P_C =$

(a) 0 (b) 1

(c) C (d) ∞

CO:5 K:2 10. The limit of a _____ sequence of analytic functions is an analytic function.

(a) uniformly convergent (b) continuous

(c) convergent (d) monotonic

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

CO:1 K:4 11. (a) State and prove Lucas' theorem.

(OR)

(b) Show that a rational function function $R(z)$ of order p has p zeros and p poles.

CO:2 K:4 12. (a) Assume that γ is the piecewise differentiable closed curve, not passing through the point a. Then prove that the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of $2\pi i$.

(OR)

(b) Analyze and prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

CO:3 K:4 13. (a) Prove that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.

(OR)

(b) State and prove Rouche's theorem.

CO:4 K:4 14. (a) Assume that u_1 and u_2 are harmonic functions in a region Ω. Prove that $\int_\gamma u_1{}^* du_2 - u_2{}^* du_1 = 0$ for every cycle γ which is homologous to zero in Ω.

(OR)

(b) Analyze the convergence of the rational function in the integral $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$ and evaluate it.

CO:5 K:4 15. (a) Assume that $f_n(z)$ is analytic in the region Ω_n, and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω. Then prove that $f(z)$ is analytic in Ω.

(OR)

(b) Assume that the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω, and $f_n(z)$ converges to $f(z)$ uniformly on every compact subset of Ω. Then prove that $f(z)$ is either identically zero or never equal to zero in Ω.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:5 16. (a) State and prove the necessary and sufficient condition for a function $f(z) = u(z) + i\, v(z)$ to be analytic.

(OR)

(b) Prove that for every power series $\sum_{n=0}^{\infty} a_n z^n$, there exists a number $R, 0 \leq R \leq \infty$ such that the series converges absolutely for every z with $|z| < R$; in $0 \leq \rho < R$ the convergence is uniform for $|z| \leq \rho$; and in $|z| > R$ the terms of the series are unbounded and the series is divergent.

CO:2 K:5 17. (a) Assume that $\varphi(\zeta)$ is continuous on the arc γ. Then prove that the function $F_n(z) = \int_\gamma \frac{\varphi(z)}{(\zeta-z)^n} d\zeta$ is analytic in each of the regions determined by γ and its derivative is $F_n{}'(z) = nF_{n+1}(z)$.

(OR)

(b) State and prove Taylor's theorem.

CO:3 K:5 18. (a) State and prove the general statement of Cauchy's theorem.

(OR)

(b) State and prove the Cauchy's Residue theorem.

CO:4 K:5 19. (a) Determine $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$ for $a > 1$.

(OR)

(b) Suppose that $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \leq R$. Then prove that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z) d\theta$ for all $|a| < R$.

CO:5 K:5 20. (a) Prove that the function $P_U(z)$ is harmonic for $|z| < 1$ and $\lim_{z \to e^{i\theta_0}} P_U(z) = U(\theta_0)$ provided that U is continuous at θ_0.

(OR)

(b) Explain the Laurent series development of $f(z)$.