

KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages)

Reg. No:.....

Question Code: 26E00807

Course Code : 24PMMA33

PG Degree - End Semester Examinations, April 2026

Third Semester
M.Sc., MATHEMATICS

Topology

(For those who joined in July 2024 onwards)

Time : 3Hours

Maximum : 75 Marks

PART - A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

- CO:1 1. In the subspace $[0,1] \cup (2,3)$ of \mathbb{R} , the set $[0,1]$ is
K:2 (a) open (b) closed
(c) both open and closed (d) neither open nor closed
- CO:1 2. Every finite point set in a Hausdorff space is
K:1 (a) open (b) closed
(c) both open and closed (d) neither open nor closed
- CO:2 3. The inverse image of a closed set is closed under a _____ map.
K:2 (a) bijective (b) continuous
(c) one - one (d) onto
- CO:2 4. The metric ρ defined by $\rho(x, y) = \max\{|x_1 - y_1|, \dots, |x_n - y_n|\}$ in
K:1 \mathbb{R}^n is called _____ the metric.
(a) euclidean (b) standard
(c) square (d) uniform
- CO:3 5. If the only subsets of a space X which are both open and closed in
K:1 X are the empty set and X itself, then X is
(a) regular (b) compact
(c) normal (d) connected
- CO:3 6. A space X is locally path connected if and only if for every open set
K:2 U of X , each path component of U is _____ in X .
(a) open (b) closed
(c) compact (d) normal

- CO:4 7. The real line \mathbb{R} is
K:2
- (a) compact but not locally compact (b) locally compact but not compact
(c) both compact and locally compact (d) neither compact nor locally compact
- CO:4 8. If every infinite subset of a space X has a limit point, then X is
K:1
- (a) compact (b) limit point compact
(c) sequentially compact (d) locally compact
- CO:5 9. A space for which every open covering contains a countable sub
K:1 covering is called a ____ space.
- (a) compact (b) regular
(c) normal (d) Lindelöf
- CO:5 10. A space X is said to be _____ if for each pair x, y of distinct points
K:1 of X , there exists disjoint open sets containing x and y respectively.
- (a) Hausdorff (b) regular
(c) normal (d) completely regular

PART - B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 250 words.

- CO:1 11. (a) Assume that \mathcal{C} is a collection of open sets of a topological
K:4 space X such that for each open set U of X and each x in U , there is an element C of \mathcal{C} such that $x \in C \subset U$. Then prove that \mathcal{C} is a basis for the topology of X .

(OR)

- (b) Assume that Y is a subspace of a topological space X . Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y .

- CO:2 12. (a) State and prove the pasting lemma for continuous function.
K:4

(OR)

- (b) State and prove the Uniform limit theorem.

- CO:3 13. (a) Prove that the union of a collection of connected subspaces
K:4 of X that have a point in common is connected.

(OR)

(b) Prove that the components of a space X are connected disjoint subspaces of X whose union is X , such that each nonempty connected subspace of X intersects only one of them.

CO:4 14. (a) Prove that the image of a compact space under a continuous map is compact.
K:4

(OR)

(b) Assume that X is locally compact Hausdorff and A is a subspace of X . If A is closed in X or open in X , then prove that A is locally compact.

CO:5 15. (a) Assume that X has a countable basis, then prove that every open covering of X contains a countable subcollection covering X .
K:4

(OR)

(b) Prove that every subspace of a Hausdorff space is Hausdorff.

PART - C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 600 words.

CO:1 16. (a) Define the product topology and prove the collection

K:5 $S = \{\pi_1^{-1}(U) / U \text{ open in } X\} \cup \{\pi_2^{-1}(V) / V \text{ open in } Y\}$ is a sub basis for the product topology on $X \times Y$.

(OR)

(b) Define the subspace topology. Further if A is a subspace of X and B is a subspace of Y , prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

CO:2 17. (a) Let $\{X_\alpha\}$ be an indexed family of spaces; let $A_\alpha \subset X_\alpha$ for each α . If $\prod X_\alpha$ is given either the product or the box topology, then prove that $\prod \bar{A}_\alpha = \overline{\prod A_\alpha}$.
K:5

(OR)

(b) Prove that the topologies on \mathbb{R}^n induced by the euclidean metric d and the square metric ρ are the same as the product topology on \mathbb{R}^n .

C0:3 18. (a) Prove that the image of a connected space under a
K:5 continuous map is connected.

(OR)

(b) Prove that a space X is locally connected if and only if for every open set U of X , each component of U is open in X .

C0:4 19. (a) Prove that every compact subspace of a Hausdorff space is
K:5 closed.

(OR)

(b) Prove that in a metrizable space, limit point compactness implies sequentially compactness.

C0:5 20. (a) Prove that every regular space with a countable basis is
K:5 normal.

(OR)

(b) State and prove Urysohn lemma.