KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:……**

Question Code: 26E00809 **Course Code : 24PMMA34**

PG Degree - End Semester Examinations, April 2026

Third Semester

M.Sc., MATHEMATICS

Calculus of Variations and Integral Equations

(For those who joined in July 2024 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:1 1. What happens if F does not contain y explicitly?

(a) F = constant (b) $\partial F/\partial y'$ = constant

(c) y = constant (d) y' = 0

CO:1 K:1 2. The necessary condition for extremum of a functional

(a) $\delta J = 1$ (b) $\delta J = 0$

(c) $dJ/dx = 0$ (d) F = 0

CO:2 K:1 3. The Euler–Lagrange equation is derived from:

(a) Newton's Second Law (b) Hamilton's Principle

(c) Gauss Law (d) Ohm's Law

CO:2 K:1 4. In a Sturm–Liouville problem, eigen functions corresponding to different eigenvalues are:

(a) Parallel (b) Orthogonal

(c) Equal (d) Zero

CO:3 K:2 5. A Volterra integral equation is characterized by:

(a) Fixed limits of integration (b) Variable upper limit of integration

(c) Infinite limits only (d) No kernel

CO:3 K:1 6. The kernel K(x, t) in an integral equation represents:

(a) Unknown function (b) Constant value

(c) Interaction between variables (d) Boundary condition

CO:4 K:1 7. A linear equation in a cause and effect relationship represents:

(a) Algebraic relation only (b) Relationship between input and output

(c) Nonlinear behaviour (d) Random function

CO:4 K:1 8. A separable kernel in a Fredholm equation is of the form:

(a) K(x, t) = f(x)g(t) (b) K(x, t) = x + t

(c) K(x, t) = constant (d) K(x, t) = 0

CO:5 K:2 9. In integral operator (Tf)(x)=∫_a^b K(x,t)f(t)dt is called a Hilbert–Schmidt operator if

(a) K(x,t) is continuous (b) ∫_a^b |K(x,t)|dt < ∞

(c) ∫_a^b∫_a^b $|K(x,t)|^2$ dxdt < ∞ (d) K(x,t) is bounded

CO:5 K:1 10. The Neumann series converges if

(a) Operator is compact (b) Operator is self-adjoint

(c) |λ| ||T|| < 1 (d) Kernel is symmetric

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

CO:1 K:4 11. (a) Analyze the Euler–Lagrange equation for

$J[y] = \int_{x_1}^{x_2} F(x, y, y') \, dx.$

(OR)

(b) Prove that if F does not contain x explicitly, then $F - y' (\partial F/\partial y') =$ constant.

CO:2 K:3 12. (a) Construct the Euler–Lagrange equation from Hamilton's principle.

(OR)

(b) Solve the method of Lagrange multipliers with an example.

CO:3 K:3 13. (a) Construct an integral equation and classify it into different types with examples.

(OR)

(b) Establish the relationship between differential equations and integral equations.

CO:4 K:3 14. (a) Analyse cause – effect relationship using linear equations with suitable example.

(OR)

(b) Construct influence function and explain its properties.

CO:5 K:3 15. (a) Construct a Hilbert–Schmidt operator and state its properties.

(OR)

(b) Solve the Fredholm alternative.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:5 16. (a) Justify the curve that minimizes

$J[y] = \int_{x_1}^{x_2} \sqrt{1 + y'}\, dx$. and prove that the shortest distance between two points is a straight line.

(OR)

(b) Value the generalized Euler–Lagrange equation when

$J[y] = \int F(x,y,y',y'')\, dx$.

CO:2 K:6 17. (a) Discuss variable end-point problems and derive the transversality condition.

(OR)

(b) Discuss Sturm–Liouville theory in detail and prove orthogonality of eigenfunctions.

CO:3 K:4 18. (a) Analyze the integral equation corresponding to a second-order linear differential equation.

(OR)

(b) Inspect the construction of Green's function for a second-order boundary value problem.

CO:4 K:3 19. (a) Identify the formulation of linear equations in cause and effect and explain the concept of influence function in detail.

(OR)

(b) Solve a Fredholm integral equation with separable kernel and explain each step clearly.

CO:5 20. (a) Prove that a Hilbert–Schmidt operator is compact.

K:5

(OR)

(b) Estimate Neumann series solution and convergence.