KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No: ……………………………………………**

Question Code: 26E00811 **Course Code: 24PMMA41**

PG Degree - End Semester Examinations, April 2026

Fourth Semester

M.Sc., MATHEMATICS

Advanced Algebra – II

(For those who joined in July 2024 onwards)

Time: 3Hours **Maximum: 75 Marks**

PART – A ($10 \times 1 = 10$ Marks)

Answer ALL Questions

Choose the correct answer:

CO:1 K:1 1. If L is a finite extension of F and K is a subfield of L which contains F, then ________.

(a) $[L:F] | [K:F]$ (b) $[K:F] | [L:F]$

(c) $[K:F] = [L:F]$ (d) $[K:F] \neq [L:F]$

CO:1 K:1 2. If a and b in K are algebraic over F of degrees m and n respectively, then $a \pm b$ is algebraic over F of degree atmost _______.

(a) $m+n$ (b) $m-n$

(c) mn (d) m^n

CO:2 K:1 3. The element $a \in K$ is a root of $p(x) \in F[x]$ of multiplicity m if

(a) $(x-a) \ |p(x)$ (b) $(x-a)^m \in p(x)$

(c) $(x-a)^m \subseteq p(x)$ (d) $(x-a)^m | p(x)$

CO:2 K:1 4. If $f(x) \in F[x]$ is irreducible and the characteristic of F is 0, then $f(x)$

(a) has no multiple roots (b) has no roots

(c) has only one root (d) has equal roots

CO:3 K:2 5. If G is a group of automorphism of K, then the fixed field of G is the set of all elements $a \in K$ such that ________ .

(a) $\sigma(a) \neq a$ for all $\sigma \in G$ (b) $\sigma(a) = a$ for all $\sigma \in G$

(c) $\sigma(a) \neq a$ for only one $\sigma \in G$ (d) $\sigma(a) = a$ for only one $\sigma \in G$

CO:3 K:2 6. The group of all automorphisms of K, $G(K,F)$ is a/an _______.

(a) Abelian group (b) Normal Subgroup

(c) Subgroup (d) Cyclic

CO:4 K:2 7. If the finite field F has p^m elements, then every $a \in F$ satisfies

(a) $a^m = a$ (b) $a^m \neq a$

(c) $a^{p^m} \neq a$ (d) $a^{p^m} = a$

CO:4 K:2 8. The multiplicative group of nonzero elements of a finite field is

(a) Cyclic (b) Subgroup

(c) Abelian (d) Normal

CO:5 K:2 9. A division ring D is said to be algebraic over a field F if

(a) F does not contain in the center of D (b) F contains in the center of D

(c) F does not contain in the edge of D (d) F contains in the edge of D

CO:5 K:2 10. The only irreducible polynomials over the field of real numbers are of degree

(a) – 1 (b) 3

(c) 1 or 2 (d) 0

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 250 words.

CO:1 K:3 11. (a) The elements in K which are algebraic over F form a subfield of K - Interview.

(OR)

(b) If L is an algebraic extension of K and K is an algebraic extension L of F, then construct an algebraic extension of F

CO:2 K:3 12. (a) Construct a proof for the statement that a polynomial of degree n over a field can have at most n roots in any extension field.

(OR)

(b) Build a proof for statement: The polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

CO:3 K:3 13. (a) If K is a field and if $\sigma_1,...,\sigma_n$ are distinct automorphisms of K, then develop an argument which shows that it is impossible to find elements $a_1,...,a_n$, not all 0, in K such that $a_1\sigma_1(u)+a_2\sigma_2(u)+...+a_n\sigma_n(u)=0$ for all $u \in K$.

(OR)

(b) Construct a proof that K is a normal extension of F if and only if K is the splitting field of some polynomial over F in detail.

CO:4 K:4 14. (a) Assume that p is a prime number and m is any positive integer, show that there exists a field having p^m elements.

(OR)

(b) Examine that the multiplicative group of nonzero elements of a finite field is cyclic.

CO:5 K:4 15. (a) Inspect the statement: Let ₵ be the field of complex numbers and suppose that the division ring D is algebraic over ₵. Then D = ₵.

(OR)

(b) Assume L is a left ideal of H. Then show that there exists an element $u \in L$ such that every element in L is a left-multiple of u.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:3 16. (a) If L is a finite extension of K and if K is a finite extension of F, then construct an argument to show that L is a finite extension of F. Moreover, $[L:F]=[L:K][K:F]$

(OR)

(b) Apply the concept of finite extension to show that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

CO:2 K:4 17. (a) If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F, then analyze and prove that there is an extension E of F, such that $[E:F]=n$, in which $p(x)$ has a root.

(OR)

(b) If F is of characteristic 0 and if a,b are algebraic over F, then inspect that there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

CO:3 K:4 18. (a) If K is a finite extension of F, then examine that $G(K,F)$ is a finite group and its order, $o(G(K,F))$ satisfies $o(G(K,F)) \leq [K:F]$.

(OR)

(b) Let K be a normal extension of F and let H be a subgroup of $G(K,F)$; let $K_H = \{x \in K \mid \sigma(x) = x \text{ for all } \sigma \in H\}$ be the fixed field of H. Then test the following (i) $[K:K_H] = o(H)$ (ii) $H = G(K,K_H)$

CO:4 K:5 19. (a) Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n. Then decide whether G is cyclic.

(OR)

(b) Justify the statement "a finite division ring is necessarily a commutative field."

CO:5 K:6 20. (a) Let D be a division ring algebraic over F, the field of real numbers. Then discuss the following: D is isomorphic to one of the field of real numbers, the field of complex numbers, or the division ring of real quaternions.

(OR)

(b) Elaborate that every positive integer can be expressed as the sum of squares of four integers.