KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No: ....**

Question Code: 26E00812 **Course Code : 24PMMA42**

PG Degree - End Semester Examinations, April 2026

Fourth Semester

M.Sc., MATHEMATICS

Functional Analysis

(For those who joined in July 2024 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:1 1. The norm of the functional f is defined as

(a) $\|f\| = inf\{|f(x)|: \|x\| \leq 1\}$ (b) $\|f\| = inf\{|f(x)|: \|x\| = 1\}$

(c) $\|f\| = sup\{|f(x)|: \|x\| \leq 1\}$ (d) $\|f\| = sup\{|f(x)|: \|x\| = 1\}$

CO:1 K:1 2. Let M be a linear subspace of a normed linear space N, and let f be a functional defined on M. If x_0 is a vector not in M, then the linear subspace spanned by M and x_0 is represented as

(a) $M_0 = M * [x_0]$ (b) $M_0 = M \oplus [x_0]$

(c) $M_0 = M \pm [x_0]$ (d) $M_0 = M + [x_0]$

CO:2 K:1 3. The space L_1 associated with a measure space X with measure m, with the inner product of two functions f *and* g defined by

(a) $\langle f, g\rangle = \int f(x)\overline{g(x)}\, dm(x)$ (b) $\langle f, g\rangle = \int g(x)\overline{f(x)}\, dm(x)$

(c) $\langle f, g\rangle = \int f(x)g(x)\overline{dm(x)}$ (d) $\langle f, g\rangle = \int f(x)g(x)dm(x)$

CO:2 K:1 4. Let $\{e_1, e_2, \ldots, e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, then ________.

(a) $\sum_{i=1}^{n} |\langle x, e_i\rangle|^2 = \|x\|^2$ (b) $\sum_{i=1}^{n} |\langle x, e_i\rangle|^2 \leq \|x\|^2$

(c) $\sum_{i=1}^{n} |\langle x, e_i \rangle| = \|x\|$ (d) $\sum_{i=1}^{n} |\langle x, e_i \rangle| \leq \|x\|$

CO:3 K:1 5. The adjoint operation $T \to T^*$ on $B(H)$. Then which of the property is wrong.

(a) $(T_1 + T_2)^* = T_1^* + T_2^*$ (b) $(\alpha T)^* = \bar{\alpha} T^*$

(c) $(T_1 T_2)^* = T_1^* T_2^*$ (d) $T^{**} = T$

CO:3 K:1 6. If A_1 and A_2 are self-adjoint operators on H, then their product $A_1 A_2$ is self-adjoint if

(a) $A_1 + A_2 = A_2 + A_1$ (b) $A_1 - A_2 = A_2 - A_1$

(c) $A_1 A_2 \neq A_2 A_1$ (d) $A_1 A_2 = A_2 A_1$

CO:4 K:2 7. Let T be an operator on H, if A is nonsingular then

(a) $\sigma(ATA^{-1}) = \sigma(T)$ (b) $\sigma(A^{-1}TA) = \sigma(T)$

(c) $\sigma(ATA^{-1}) = \sigma(A)$ (d) $\sigma(A^{-1}TA) = \sigma(A)$

CO:4 K:2 8. An operator T on H is normal if its _______ is a polynomial of T.

(a) Adjacent T^* (b) Adjoint T^*

(c) Factor T^* (d) Adherent T^*

CO:5 K:2 9. The weak operator topology on $B(H)$ is the weak topology generated by a function of the form

(a) $T \to \langle Tx, Ty \rangle$ (b) $T \to \langle x, Ty \rangle$

(c) $T \to \langle Tx, y \rangle$ (d) $T \to \langle x, y \rangle$

CO:5 K:2 10. The linear operators f *and* g are defined pointwise then the operation in the expression $\langle f * g \rangle(x) = \sum_{m=-\infty}^{\infty} f(n-m) g(m)$ is called

(a) Summation (b) Pointwise product

(c) Cumulation (d) Convolution

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

CO:1 K:3 11. (a) If N and N' are normed linear spaces, then construct that the set $\mathfrak{B}(N, N')$ of all continuous linear transformations of N into N' is itself a normed linear space with respect to the pointwise linear operations.

(OR)

(b) If N is a normed linear space, then develop that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

CO:2 K:4 12. (a) Analyze the statement of Uniform Boundedness Theorem and prove in detail.

(OR)

(b) If x and y are any two vectors in a Hilbert space, then examine that $|\langle x, y\rangle| \leq \|x\| \, \|y\|$.

CO:3 K:3 13. (a) If T is an operator on H for which $\langle Tx, x\rangle = 0$ for all x, then inspect that $T = 0$.

(OR)

(b) If T is an operator on H, then interview that T is normal $\Leftrightarrow$ its real and imaginary parts commute.

CO:4 K:4 14. (a) If T is normal, then analyze that the M_i's are pairwise orthogonal.

(OR)

(b) If T is normal, then examine that x is an eigen vector of T with eigen value $\lambda \Leftrightarrow x$ is an eigen vector of T^* with eigen value $\bar{\lambda}$.

CO:5 K:4 15. (a) Test for the statement that the boundary of S is a subset of Z.

(OR)

(b) If the norm in A satisfies the inequality $\|xy\| \geq K\|x\| \; \|y\|$ for some positive constant K, then simplify that A = C.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:3 16. (a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$, then construct that N/M is a normed linear space.

(OR)

(b) Let M be a linear subspace of a normed linear space N, and let f be a functional defined on M. Then construct f can be extended to a functional f_0 defined on the whole space N such that $\|f_0\| = \|f\|$.

CO:2 K:4 17. (a) Examine the statement that if B and B' are Banach spaces, and if T is a continuous linear transformation of B onto B', then T is an open mapping.

(OR)

(b) If M is a closed linear subspace of a Hilbert space H, then inspect that $H = M \oplus M^{\perp}$.

CO:3 K:4 18. (a) Let H be a Hilbert space, and let f be an arbitrary functional in H^*. Then analyze that there exists a unique vector y in H such that $f(x) = \langle x, y \rangle$ for every x in H.

(OR)

(b) If $P_1, P_2, ..., P_n$ are the projections on closed linear subspaces $M_1, M_2, ..., M_n$ of H, then examine that $P = P_1 + P_2 + ... + P_n$ is a projection $\Leftrightarrow$ the P_i's pairwise orthogonal.

CO:4 K:5 19. (a) If T is normal, then explain that the M_i's span H.

(OR)

(b) If T is an arbitrary operator on H, then justify that the eigenvalues of T constitute a non-empty finite subset of the complex plane.

CO:5 K:6 20. (a) Elaborate that the mapping $x \to x^{-1}$ of G into G is continuous and is therefore a homeomorphism of G onto itself.

(OR)

(b) Formulate that $\sigma(x)$ is non-empty.