KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:……**

Question Code: 26O03301 **Course Code : 24PMPH11**

PG Degree - End Semester Examinations, November 2026

First Semester

M.Sc. PHYSICS

Mathematical Physics

(For those who joined in July 2024 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:2 1. The eigenvalues of a Hermitian operator are always _____.

(a) Complex (b) Purely imaginary

(c) Real (d) Zero

CO:1 K:1 2. A vector space with no finite basis is said to be _____.

(a) Finite dimensional (b) Infinite dimensional

(c) Three dimensional (d) n dimensional

CO:2 K:1 3. The necessary condition for the function f(z) to be analytic at the point z is _____.

(a) $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial v}{\partial y}$ (d) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

CO:2 K:2 4. Contour integration evaluates real definite integrals by_____.

(a) Differentiating under the integral sign

(b) Choosing a closed contour and applying the Residue Theorem

(c) Using Taylor series only

(d) Substituting z = x + iy directly

CO:3 K:2 5. If A is the 3 x 3 matrix whose determinant is 4, and two eigenvalues are -1 and +2; then the third eigenvalue of the matrix is equal to ____.

(a) -2 (b) -1

(c) 1 (d) 2

CO:3 K:2 6. The matrix A = $\begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ then eigen values of A^2 are_____.

(a) -1,-9,-4 (b) 1,9,4

(c) -1,-3,2 (d) 1,3,-2

CO:4 K:1 7. The Fourier transform of F(s) is defined as _____.

(a) F(s) = $(1/2\pi)\int_{-\infty}^{\infty} f(t)e^{ist}\,dt$ (b) F(s) = $(1/2\pi)\int_{-\infty}^{\infty} f(t)e^{st}\,dt$

(c) f(x) = $(1/\sqrt{2\pi})\ \int_{-\infty}^{\infty} e^{-isx}\,F(s)ds$ (d) f(x) = $(1/\sqrt{2\pi})\ \int_{-\infty}^{\infty} e^{isx}\,F(s)ds$

CO:4 K:2 8. The Laplace transform of f(t) is _____.

(a) $\int_0^{\infty} f(t)e^{-st}\,dt$ (b) $\int_0^{\infty} f(t)e^{st}\,dt$

(c) $\int_0^{1} f(t)e^{-st}\,dt$ (d) $\int_0^{\infty} f(t)cos\,(t)\,dt$

CO:5 K:1 9. The Rodrigues formula for Legendre polynomials $P_n(x)$ is _____.

(a) $\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ (b) $\frac{1}{2^n n!}\frac{d^n}{dx^n}(x-1)^n$

(c) $\frac{1}{2^n n}\frac{d^n}{dx^n}(x-1)^n$ (d) $\frac{1}{n!}\frac{d^n}{dx^n}(x-1)^2$

CO:5 K:1 10. The Hermite differential equation is _____.

(a) $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ (b) $(1+x^2)y'' + 2xy' + n(n+1)y = 0$

(c) $y'' - 2xy' + 2ny = 0$ (d) $y'' + 2xy' + 2ny = 0$

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 250 words.

CO:1 K:3 11. (a) If $|\Psi\rangle = \begin{bmatrix} -3i \\ 2+i \\ 4 \end{bmatrix}$ and $|\varphi\rangle = \begin{bmatrix} 2 \\ -i \\ 2-3i \end{bmatrix}$ then evaluate the scalar product $\langle\varphi|\Psi\rangle$.

(OR)

(b) If $|\Psi_1\rangle = 2i|\varphi_1\rangle + |\varphi_2\rangle - a|\varphi_3\rangle + 4|\varphi_4\rangle$ and $|\Psi_2\rangle = 3|\varphi_1\rangle - i|\varphi_2\rangle + 5|\varphi_3\rangle - |\varphi_4\rangle$ are orthogonal then evaluate the value of a.

CO:2 K:4 12. (a) Show that the function $e^x(\cos x + i\, siny)$ is an analytic function and find its derivative.

(OR)

(b) Find the poles of $f(z) = \frac{1}{z^4+1}$ where z is a complex number.

CO:3 K:4 13. (a) Evaluate Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(OR)

(b) If $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$ is a 2x2 non-singular matrix, then evaluate the value of A^{-1}.

CO:4 K:3 14. (a) Find the Fourier cosine transform of $5e^{-2x} + 2e^{-5x}$.

(OR)

(b) Find the Laplace transform of $1 + \sin 2t$.

CO:5 K:4 15. (a) Using Legendre polynomials, show that $\int_{-1}^{+1} P_m(x)P_n(x) = 0$, if $m \neq n$.

(OR)

(b) Using Hermite polynomials, show that $H_{2n+1}(0) = 0$ *and* $H'_{2n}(0)=0$ if n>1.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 600 words.

CO:1 K:5 16. (a) Examine the properties of the projection operator and show that the operator $|\Psi\rangle\langle\Psi| \, is \, a$ projection operator only when$|\Psi\rangle$ is normalized.

(OR)

(b) Examine the Gram-Schmidt orthogonalization condition.

CO:2 K:5 17. (a) Use contour integration to evaluate the real integral $\int_0^\infty \frac{dx}{(1+x^2)^3}$.

(OR)

(b) Using the definition of abelian group and show that three cube roots of form an abelian group under multiplication.

CO:3 K:5 18. (a) If $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ and $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ then show that the matrices are Hermitian and Unitary matrices.

(OR)

(b) Find the eigenvectors of the matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ Then show that the eigenvectors of matrix M are linearly independent to each other.

CO:4 K:6 19. (a) Validate Dirichlet's conditions for $f(x) = x + x^2$ and deduce fouries series for $f(x)$in the limit $-\pi < x < \pi$.

(OR)

(b) Validate the existence theorem in Laplace transform and show that $L\left(\frac{1}{\sqrt{\pi t}}\right) = \frac{1}{\sqrt{s}}$ given that $L\left(2\sqrt{\frac{t}{\pi}}\right) = \frac{1}{s^{3/2}}.1$

CO:5 K:4 20. (a) Derive the first four Hermite polynomials using Rodrigues' formula.

(OR)

(b) Derive the series solution for the differential equation $y'' + y = 0$ and verify it using the analytic solution.