

KAMARAJ COLLEGE (Autonomous)

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Affiliated to Manonmaniam Sundaranar University

THOOTHUKUDI – 628 003

5 Pages

Reg. No:

Question. Code No : 2400331

Sub Code : 24PMPH11

PG Degree - End Semester Examinations, November 2024

First Semester

M.Sc. Physics

Major - MATHEMATICAL PHYSICS

(For those who joined in July 2024 onwards)

Time : 3 Hours

Maximum : 75 Marks

PART A – (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer

1. $\nabla \log r$ will be equal to

(a) $\vec{r}$

(b) $\frac{\vec{r}}{r^3}$

(c) $\frac{\vec{r}}{r^2}$

(d) $\frac{\vec{r}}{r^4}$

2. If $\vec{a}$ is a constant vector, then $\vec{r} \times \vec{a}$ is

- (a) solenoidal (b) irrotational
 (c) Both solenoidal and irrotational (d) Neither solenoidal nor irrotational
3. Taylor's series expansion of $\tan z$ about $z=0$ is
 (a) $1 - \frac{z}{3} - \frac{z^2}{18} + \dots$ (b) $1 + \frac{z}{3} - \frac{z^2}{18} + \dots$
 (c) $1 + \frac{z}{3} + \frac{z^2}{18} + \dots$ (d) $1 - \frac{z}{3} + \frac{z^2}{18} + \dots$
4. Analytic function is
 (a) Single valued (b) bounded
 (c) differentiable (d) All of these
5. If $A = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ then A^{-1} will be
 (a) $\begin{bmatrix} i & 1 \\ 0 & i \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 1 & 1 \end{bmatrix}$
 (c) $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ (d) $\begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$
6. Inverse of the matrix exists if
 (a) Matrix is singular (b) Matrix is non-singular
 (c) Matrix is Hermitian (d) Matrix is skew Hermitian
7. $L\{f(at)\}$ will be equal to
 (a) $\frac{1}{2}f\left(\frac{s}{a}\right)$ (b) $\frac{1}{a}f\left(\frac{s}{a}\right)$
 (c) $\frac{1}{a}f(s)$ (d) $af\left(\frac{s}{a}\right)$
8. The Laplace transform of $\sinh at$, is if $s > a$
 (a) $\frac{s^2 - a^2}{a}$ (b) $\frac{a}{s^2 - a^2}$
 (c) $\frac{1}{s^2 - a^2}$ (d) $\frac{a}{s^2 + a^2}$
9. The value of $P_{2m+1}(0)$ is
 (a) 0 (b) 1

- (c) $\frac{\pi}{2}$ (d) None of above
10. $H_n(-x)$ will be equal to
- (a) $(-1)^n H_n(x)$ (b) $H_n(x)$
- (c) $-H_n(x)$ (d) Zero

PART B - (5X5=25Marks)

Answer ALL Questions choosing either (a) or (b).

(Answer should not exceed 250 words.)

11. (a) Define Bra and Ket notation.
(OR)
- (b) Check whether the vectors are linearly dependent or Independent $[1,2,4], [1,0,0], [0,1,0], [0,0,1]$
12. (a) Derive Cauchy's Integral formula
(OR)
- (b) Find the residue of $f(z) = \frac{e^z}{z^2+a^2}$ at its singularities
13. (a) Find the Eigen values and Eigen values of the Matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(OR)
- (b) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$
14. (a) Give any two properties of Fourier Transform
(OR)
- (b) Find the inverse Laplace Transform of $\frac{1}{\sqrt{(2s+5)}}$
15. (a) Prove that $H_n'(x) - 2nH_{n-1}(x)$
(OR)

(b) Prove that $P_{2m+1}(0)=0$

PART C – (5 × 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).

(Answer should not exceed 600 words.)

16. (a) What is linear vector space? Give any three examples of linear vector space.

(OR)

- (b) Check the following vectors are linearly dependent or independent $(1,2,-3)$ $(2,5,1)$ $(-1,1,4)$

17. (a) Check whether the following are analytic functions of complex variable.

i) $f(z) = \sin z$ ii) $f(z) = \operatorname{Re} z$

(OR)

- (b) Define Isomorphic and homomorphic groups. Give any two properties.

18. (a) Find the characteristic equation of the following matrix and verify Cayley-Hamilton theorem.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

(OR)

- (b) Show that the

- (i) All the eigen values of a Hermitian matrix are real and
- (ii) The eigen vectors corresponding to distinct eigen values corresponding to distinct eigen values are orthogonal.

19. (a) Obtain Laplace transform of the function $f(t) = \sinh at$ at $\sin at$

(OR)

- (b) Give the properties of Inverse Laplace transform.
20. (a) Derive the orthogonal property of Hermite polynomial

(OR)

- (b) Give the orthogonal property of Legendre polynomial

$$\int_{-1}^{+1} P_m(x) P_n(x) dx = 0 \quad \text{for } m \neq n$$
$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$$