KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

THOOTHUKUDI – 628 003

(7 Pages) **Reg. No: ……………………………………………**

Question Code No : 25O00206 **Course Code : 24UMMA31**

UG Degree - End Semester Examinations, November 2025

Third Semester

B.Sc. MATHEMATICS

Vector Calculus and Its Applications

(For those who joined in July 2024 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

1. Which of the following is a vector point function?

   (a) Temperature (b) Mass

   (c) Velocity (d) Pressure

2. If $\vec{A}(t) = t\vec{i}$ and $\vec{B}(t) = t^2\vec{j}$, then $\frac{d}{dt}(\vec{A} \cdot \vec{B}) =$

   (a) $2t^2$ (b) t^2

   (c) 0 (d) t^3

3. The gradient of a scalar point function is:

(a) A scalar (b) A vector

(c) A tensor (d) A constant

4. If $\vec{A} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, then $\nabla \cdot \vec{A} =$

(a) 0 (b) 1

(c) 3 (d) $x + y + z$

5. If $\vec{F} = \nabla\vec{\phi}$ for some scalar function ϕ, then $\vec{F}$ is always:

(a) Solenoidal (b) Irrotational

(c) Path-dependent (d) Circulatory

6. If a vector field $\vec{F}$ is conservative, then the line integral of $\vec{F}$ over a closed path is:

(a) Zero (b) Non-zero

(c) Does not exist (d) Infinite

7. The volume integral of a scalar function $\phi(x, y, z)$ over a volume V is denoted by:

(a) $\iint_S \phi \, dS$ (b) $\iiint_V \phi \, dV$

(c) $\oint_C \phi \, ds$ (d) $\nabla \times F$

8. The infinitesimal surface element vector $d\vec{S}$ is:

(a) $\hat{n}\, dS$ (b) $dx\, dydz$

(c) $ds\, \hat{t}$ (d) $\nabla\phi$

9. The divergence theorem relates:

(a) Line integral to surface integral

(b) Surface integral to volume integral

(c) Gradient to curl

(d) Curl to divergence

10. Which theorem can be applied to calculate the area of a planar region using a line integral?

(a) Gauss Divergence Theorem

(b) Stokes' Theorem

(c) Green's Theorem

(d) Fundamental Theorem of Line Integrals

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

11. (a) If ϕ is a scalar function of u and "a" a constant vector, then show that $\frac{d(\phi a)}{du} = a\frac{d\phi}{du}$.

(OR)

(b) Find the derivatives of $\vec{A} \times \vec{B}$ with respect to u, if $\vec{A} = 2u\,\vec{i} + u^2\,\vec{j}$ and $\vec{B} = -u\,\vec{j} + \vec{k}$.

12. (a) Prove that $div\,(r^n\vec{r}) = (n+3)r^n$given that $\vec{r} = \overrightarrow{xi} + \overrightarrow{yj} + \overrightarrow{zk}$.

(OR)

(b) If $\varphi = x + xy^2 + yz^3$, find $\nabla\varphi$ at (0,1,1).

13. (a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$

(OR)

(b) Find the value of the line integral $\int c\vec{f}.d\vec{r}$ where $\vec{F} = y\vec{\imath} + x\vec{\jmath}$ from $(0,0)$ to $(1,1)$.

14. (a) Evaluate the integral $\iint_S \vec{A}.n\,dS$ if $\vec{A} = 4y\,\vec{\imath} + 18z\,\vec{\jmath} - x\,\vec{k}$ and S is the surface of the portion of the plane $3x + 2y + 6z = 6$ contained in the first octant.

(OR)

(b) Evaluate $\iiint_V \nabla.\vec{F}\ dV$ if $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ and V is the volume of the region enclosed by the cube $0 \le x, y, z \le 1$.

15. (a) Show that $\iint_S \vec{r}.n\,dS = 4\pi a^3$, if S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

(OR)

(b) If $\vec{f} = (2y+3)\vec{\imath} + xz\,\vec{\jmath} + (yz - x)\vec{k}$, evaluate $\int \vec{f}.d\vec{r}$ along the following path C given by $x = 2t^2, y = t, z = t^3$ from $t = 0$ to $t = 1$.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 500 words.

16. (a) If A, B, C are functions of the scalar variable u, prove that

i. $\frac{d}{du}[ABC] = \left[\frac{dA}{du}, B, C\right] + \left[A, \frac{dB}{du}, C\right] + \left[A, B, \frac{dC}{du}\right]$

ii. $\frac{d}{du}\{A \times (B \times C)\} = \frac{dA}{du} \times (B \times C) + A \times \left(\frac{dB}{du} \times C\right) + A \times \left(B \times \frac{dC}{du}\right)$

(OR)

(b) Find the derivatives of $\vec{A}.\vec{B}$ and $\vec{A} \times \vec{B}$ with respect to u if $\vec{A} = u^2\,\vec{i} + u\,\vec{j} + 2u\,\vec{k}$ and $\vec{B} = \vec{j} - u\,\vec{k}$.

17. (a) If $\nabla\varphi = (y + y^2 + z^2)\vec{i} + (x + z + 2xy)\vec{j} + (y + 2zx)\vec{k}$ and if $\varphi(1,1,1) = 3$, find φ.

(OR)

(b) Given that $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = r$, show that

i. $\nabla\left(\frac{1}{r}\right) = -\frac{r}{r^3}$

ii. $\nabla f(r) = f'(r)\hat{r}$.

18. (a) Find the value of the integral $\int_C \vec{A}.d\vec{r}$, where $\vec{A} = yz\,\vec{i} + zx\,\vec{j} - xy\,\vec{k}$ where C is the curve obtained by joining O to

$A(2,0,0)$, then to $B(2,4,0)$ and then to $Q(2,4,8)$ by straight lines.

(OR)

(b) Show that the integral of $\vec{F} = (3x^2 + 6xy)\vec{i} + (3x^2 - y^3)\vec{j}$ is independent of the path of integration. Find $\int \vec{f}.\overrightarrow{dr}$ along any curve joining (0,0) and (1,2).

19. (a) Evaluate $\iint_S \vec{A}.n\ ds$ if $\vec{A} = z\,\vec{i} + x\vec{j} - y^2 z\,\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ contained in the first octant between the planes $z = 0$ and $z = 2$.

(OR)

(b) Evaluate $\iiint_V F\ \ dV$, where $\vec{F} = 2xz\,\vec{i} - x\,\vec{j} + y^2\,\vec{k}$ and V is the volume of the region enclosed by the cylinder $x^2 + y^2 = a^2$ between the planes $z = 0, z = c$.

20. (a) Check Gauss Divergence theorem for the vector function $\vec{F} = 4x\,\vec{i} - 2y^2\vec{j} + z^2\,\vec{k}$ over the cylinder bounded by $x^2 + y^2 = 4, z = 0, z = 3$.

(OR)

(b) Verify Strokes' theorem for $\vec{A} = (y - z + 2)\vec{\imath} + (yz + 4)\vec{\jmath} - xz\,\vec{k}$, where S is the surface of the cube x=0, x=2, y=0, y=2, z=0, z=2 above the xoy plane.