KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:……………………………………………**

Question Code: 26E00802 **Course Code : 25PEMA11**

PG Degree - End Semester Examinations, April 2026

First Semester

M.Sc., MATHEMATICS

Graph Theory and Applications

(For those who joined in June 2025 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:1 1. A 3 –regular graph is called _________ graph.

(a) Cubic (b) Spanning

(c) Hyper (d) Isomorphic

CO:1 K:2 2. The union of two distinct paths joining two distinct vertices contains a

(a) Cycle (b) Triangle

(c) Longest path (d) Edge

CO:2 K:1 3. The connectivity and edge connectivity of a simple cubic graph G are _____

(a) Unequal (b) Equal

(c) Cubic (d) Finite

CO:2 K:2 4. A ____________ of a graph G is maximal non separable sub graph of G.

(a) Path (b) Cycle

(c) Block (d) Sub graph

CO:3 K:1 5. The _________ of a vertex u of T is the maximum number of edges in any branch at u

(a) Centroid (b) weight

(c) Eccentricity (d) branch

CO:3 K:2 6. A vertex v is called central vertex if

(a) $e(v) > r(G)$ (b) $e(v) < r(G)$

(c) $e(v) = r(G)$ (d) $e(v) \neq r(G)$

CO:4 K:2 7. A ________ of G is set of independent edges.

(a) Matching (b) Spanning

(c) complement (d) Connected set

CO:4 K:1 8. Every connected 3-regular graph having no cut edge has

(a) 1-factor (b) 2-factor

(c) 3-factor (d) 4-factor

CO:5 K:1 9. A ________ in a graph G is a spanning trail that contains all the edges of G.

(a) Hamiltonian walk (b) Euler Trial

(c) Euler Walk (d) Hamiltonian trial

CO:5 K:2 10. Every k-critical graph is ________ edge connected.

(a) k (b) k-1

(c) k-2 (d) k-3

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

CO:1 K:3 11. (a) Define graph isomorphism and self complementary graph with suitable example.

(OR)

(b) Prove that in a simple graph G, if G is not connected, then G^c is connected.

CO:2 K:2 12. (a) Summarize the proof of the following "A vertex v of a connected graph G with at least 3 vertices is a cut vertex of G if and only if there exists vertices u and w of G distinct from v such that v is in every u-w path".

(OR)

(b) Define r-connected. Prove that a graph G is at least three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.

CO:3 K:4 13. (a) Discover that a simple graph is a tree if and only if every two distinct vertices are connected by unique path.

(OR)

(b) Illustrate the proof of the statement: Every tree has a center consisting of either a single vertex or two adjacent vertices.

CO:4 K:4 14. (a) Make use of the definitions of independent set and covering to prove that $\alpha + \beta = n$.

(OR)

(b) Determine whether in a bipartite graph G, the minimum number of vertices that cover all the vertices of G is equal to maximum number of independent edges.

CO:5 K:3 15. (a) If G is Hamiltonian, then prove that $\omega(G - S) \leq |S|$, for every non-empty proper subset S of V.

(OR)

(b) Prove that, $\delta(G) \geq k - 1$, when G is k-critical.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:3 16. (a) Define line graph of G, provide example and prove that a line graph of a simple graph is path if and only if G is a path.

(OR)

(b) Elaborately explain various types of graph products with suitable example.

CO:2 K:4 17. (a) By developing the proof of $\kappa(G) \leq \lambda(G) \leq \delta(G)$, give an example for a graph in which equality holds strict.

(OR)

(b) Demonstrate that in a 2-connected graph G any two longest cycles have at least two vertices in common.

CO:3 K:4 18. (a) Prove that the number of edges in a tree on n vertices is n-1. Conversely connected graph on n vertices and n-1 edges is a tree.

(OR)

(b) By defining contraction of a graph with example, prove that $\tau(G) = \tau(G - e) + \tau(G.e)$ where e is not a loop of a connected graph G.

CO:4 K:4 19. (a) Paraphrase that, a matching M of a graph G is maximum if and only if it has no M-augmenting path.

(OR)

(b) Show that if G is a bipartite graph with bipartition (X,Y) then G contains a matching M that saturates all the vertices of X if and only if $|N(S)| \geq |S|$, for every subset S of X.

CO:5 K:4 20. (a) What is meant by closure of a graph? Prove that C(G) is well defined.

(OR)

(b) Defend the proof of Brooke's theorem.