KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:…………………………………………**

Question Code: 26E00803 **Course Code : 24PMMA21/25PMMA21**

PG Degree - End Semester Examinations, April 2026

Second Semester

M.Sc., MATHEMATICS

Ring Theory and Lattices

(For those who joined in July 2024 and June 2025 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:1 1. If F is a field, then find the only ideals of F ________

(a) (0) and F (b) F only

(c) (0) only (d) None of these

CO:2 K:1 2. If U is an ideal of R and $1 \in U$, then_______.

(a) $U = R$ (b) R/U

(c) U/R (d) None of these

CO:2 K:1 3. If $a|b$ and $b|c$, then________

(a) $b = c$ (b) $b|c$

(c) $b|a$ (d) $a|c$

CO:3 K:1 4. If $a|b$ and $a|c$, then which of the following is true?

(a) $a|bc$ (b) $a|(b \pm c)$

(c) $b|a$ (d) None of these

CO:1 K:1 5. The polynomial $f(x) = a_0 + a_1x + \cdots + a_nx^n$, where the $a_0, a_1, a_2, \cdots, a_n$ are integers is said to be primitive if the greatest common divisor of $a_0, a_1, a_2, \cdots, a_n$ is _____.

(a) 1 (b) 0

(c) 2 (d) None of these

CO:5 K:1

6. A polynomial $p(x)$ in $F[x]$ is said to be irreducible over F whenever $p(x) = a(x)b(x)$ with $a(x), b(x) \in F[x]$, then one of $a(x)$ or $b(x)$ has degree ______.

   (a) 1 (b) 2

   (c) 0 (d) None of these

CO:4 K:1

7. The only idempotent element in $rad\ R$ is ______

   (a) 2 (b) 1

   (c) R (d) 0

CO:3 K:1

8. For any ring R, the quotient ring $R/rad\ R$ is semi simple; that is $rad(R/radR) =$______

   (a) $\{0\}$ (b) 1

   (c) $rad\ R$ (d) None of these

CO:4 K:1

9. If $a \geq b$, then which of the following is true?

   (a) $d(a) = d(b) + length\ I[b, a]$

   (b) $d(a) = d(b) - length\ I[b, a]$

   (c) $d(a) = length\ I[b, a]$

   (d) $d(b) = length\ I[b, a]$

CO:5 K:1

10. Identify the greatest and least elements of a Boolean algebra from the choices below.

   (a) $0, -1$ (b) $0,1$

   (c) $-1,2$ (d) $1,1$

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 250 words.

CO:5 K:4

11. (a) If R is a commutative ring with a unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

(OR)

   (b) Prove that every integral domain can be imbedded in a field.

CO:1 K:4

12. (a) State and prove Fermat's Theorem.

(OR)

   (b) Prove that an Euclidean ring possess a unit.

CO:2 K:4 13. (a) If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)g(x)$ is a primitive polynomial.

(OR)

(b) If $f(x), g(x)$ are two non zero elements of $F[x]$, then prove that $\deg\big(f(x)g(x)\big) = \deg f(x) + \deg g(x)$

CO:3 K:4 14. (a) If R/I is a semi simple ring then prove that $rad\ R \subseteq I$.

(OR)

(b) Prove that For any ring R, the quotient ring $R/Rad\ R$ has no prime radical.

CO:4 K:4 15. (a) Prove that the lattice of normal subgroups of a group is modular and the lattice of submodules of a module is modular.

(OR)

(b) Prove that A bijective map of a lattice L into a lattice L' is a lattice isomorphism if and only if it and its inverse are order preserving.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:4 K:6 16. (a) Suppose let R be a commutative ring with a unit element whose only ideals are (0) and R itself. Then prove that R is a field.

(OR)

(b) Suppose if ϕ is a homomorphism of R into R'. Then prove that

i. $\phi(0) = 0$
ii. $\phi(-a) = -\phi(a)$ for every $a \in R$.

CO:5 K:5 17. (a) Whether $J[i]$ is a Euclidean ring or not? Justify.

(OR)

(b) Let R be a Euclidean ring. Then prove that every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

CO:3 18. (a) State and prove the Division algorithm.

K:6

(OR)

(b) State and prove the Eisenstein criterion.

CO:1 19. (a) If I is an ideal of the ring R, then prove that

K:5

i. $rad\left(\frac{R}{I}\right) \supseteq \frac{radR+I}{I}$ and

ii. Whenever $I \subseteq rad\ R, rad(R/I) = (rad\ R)/I$.

(OR)

(b) Let R be a principal ideal domain. Then prove that R is semi simple if and only if R is either a field or has an infinite number of maximal ideals.

CO:2 20. (a) Prove that a lattice L is modular if and only if whenever

K:5

$a \geq b$ and $a \wedge c = b \wedge c$ and $a \vee c = b \vee c$ for some in L, then $a = b$.

(OR)

(b) If a and b are elements of a modular lattice, then prove that the map $x \to x \wedge b$ is an isomorphism of the interval

$I[a, a \vee b]$ onto $I[a \wedge b, b]$. The inverse isomorphism is $y \to y \vee a$.