KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages) **Reg. No:…………………………………**

Question Code: 26E00804 **Course Code: 24PMMA22/25PMMA22**

PG Degree - End Semester Examinations, April 2026

Second Semester

M.Sc., MATHEMATICS

Real Analysis II

(For those who joined in July 2024 and June 2025 onwards)

Time : 3Hours **Maximum : 75 Marks**

PART – A ($10 \times 1 = 10$ Marks)

Answer ALL Questions

Choose the correct answer :

CO:1 K:1

1. If $m^*(E) = 0$ then E is ________.
   (a) σ-algebra (b) Measurable
   (c) Borel set (d) Borel function

CO:1 K:2

2. If f is a measurable function and esssup $|f| < \infty$, then f is said to be ________.
   (a) Essentially bounded (b) Essential supremum
   (c) Essential infinum (d) Borel function

CO:2 K:1

3. Find the value $\int_1^\infty \frac{dx}{x} =$ ________.
   (a) 0 (b) −1
   (c) 1 (d) ∞

CO:2 K:2

4. If f and g are measurable $|f| \leq |g|$ a. e. and g is integrable, then f is ___________.
   (a) Disjoint measurable (b) Positive
   (c) Integrable (d) Negative

CO:3 K:1

5. A function f is said to be ________ with period $p \neq 0$ if f is defined on R and if $f(x+p) = f(x)$ for all x.
   (a) Periodic (b) Fourier series
   (c) Riemann-Lebesgue (d) Consequences

CO:3 K:2

6. If the limit $s(x)$ exists and if the Lebesgue integral $\int_0^{\delta} \frac{g(t)-s(x)}{t} dt$ exists for some $\delta < \pi$, then the Fourier series generated by f ________ to $s(x)$.

(a) Integral (b) Periodic

(c) Converges (d) Bounded

CO:4 K:1

7. The function f is said to be differentiable at c if there exists a ________ $T_c: R^n \to R^m$ such that $f(u+v) = f(c) + T_c(v) + \|v\| E_c(v)$.

(a) Differentiable (b) Linear function

(c) Continuous (d) Gradient vector

CO:4 K:2

8. Choose the $f'(x,t) =$ ________.

(a) $\sum_{i=1}^{n} D_i f(x) t_i$ (b) $\sum_{i=1}^{n} (D_i + f(x) t_i)$

(c) $\sum_{i=1}^{n} (D_i - f(x) t_i)$ (d) $\sum_{i=1}^{n} \left(\frac{D_i}{f(x) t_i}\right)$

CO:5 K:1

9. If $f = u + iv$ is a complex-valued function with a derivative at a point z in c, then $J_f(z) =$________.

(a) $f(z)$ (b) $|f'(z)|$

(c) $|f''(z)|$ (d) $|f'(z)|^2$

CO:5 K:2

10. If $\Delta < 0$, f has a ________ point at a.

(a) Saddle (b) Minimum

(c) Maximum (d) Zero

PART – B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 250 words.

CO:1 K:3

11. (a) For any sequence of sets $\{E_i\}$, Examine that $m^*(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(E_i)$.

(OR)

(b) Let $\{f_n\}$ be a sequence of measurable functions defined on the same measurable set. Then Develop (i) $\sup_{1 \leq i \leq n} f_i$ is measurable for each n (ii) $\inf_{1 \leq i \leq n} f_i$ is measurable for each n (iii) $\sup f_n$ is measurable (iv) $\inf f_n$ is measurable.

CO:2 K:4

12. (a) Show the Proof of Lebesgue's Dominated Convergence theorem.

(OR)

(b) Let f be a bounded measurable function defined on the finite interval (a, b). Examine that $\log_{\beta\to\infty} \int_a^b f(x)\sin\beta x\,dx = 0$.

CO:3 K:3 13. (a) State and Prove Bessel's inequality.

(OR)

(b) Assume that $f \in L(I)$. Then show that for each real β, we have $\lim_{\alpha\to+\infty} \int_I f(t)\,sin(\alpha t + \beta)dt = 0$.

CO:4 K:4 14. (a) If f is differentiable at c, then examine that f is continuous at c.

(OR)

(b) Analyze Mean-Value theorem.

CO:5 K:3 15. (a) Assume that $f = (f_1, f_2, \dots, f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S, then identify there is a n-ball B(a) on which f is one-to-one.

(OR)

(b) A quadric surface with center at the origin has the equation $Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$. Solve the lengths of its semi-axes.

PART – C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).
Answer should not exceed 600 words.

CO:1 K:3 16. (a) Show that the outer measure of an interval equals its length.

(OR)

(b) Prove that the class M is a σ-algebra.

CO:2 K:4 17. (a) Classify Fatou's lemma.

(OR)

(b) Let f and g be integral functions. Analyze (i) af is integrable and $\int af\,dx = a\int f\,dx$ (ii) f + g is integrable and $\int (f+g)dx = \int f\,dx + \int g\,dx$ (iii) If f = 0 a.e., then $\int f\,dx = 0$ (iv) If $f \leq g$ a.e., then $\int f\,dx \leq \int g\,dx$ (v) If A and B are disjoint measurable sets, then $\int_A f dx + \int_B f dx = \int_{A \cup B} f dx$.

CO:3 18. (a) State and prove the Jordan theorem.

K:5

(OR)

(b) Interpret Fejer theorem.

CO:4 19. (a) Assume that one of the partial derivatives $D_1 f, \ldots, D_n f$ exists at c and that the remaining n–1 partial derivatives exist in some n-ball B(c) and are continuous at c. then justify f is differentiable at c.

K:5

(OR)

(b) Develop Taylor's formula.

CO:5 20. (a) Elaborate Implicit function theorem.

K:6

(OR)

(b) Formulate Second-derivative test for extrema.