

KAMARAJ COLLEGE (Autonomous)

Accredited with A+ Grade by NAAC

(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages)

Reg. No:.....

Question Code: 26E00212

Course Code: 24UEMA21/25UEMA21

UG Degree - End Semester Examinations, April 2026

Second Semester
MATHEMATICS ALLIED

Allied Mathematics II - Vector Calculus and Fourier Series
(For those who joined in July 2024 and June 2025 onwards)

Time : 3Hours

Maximum : 75 Marks

PART - A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

- CO:1 1. In a vector calculus, the equation of the tangent plane is _____.
K:1 (a) $(\vec{r} - \vec{r}_0) \times \nabla\phi = 0$ (b) $(\vec{r} + \vec{r}_0) \cdot \nabla\phi = 0$
(c) $(\vec{r} - \vec{r}_0) \cdot \nabla\phi = 0$ (d) $(\vec{r} - \vec{r}_0) \cdot \phi = 0$
- CO:1 2. If the vector $\vec{F} = (kx + 3y + 4z)\vec{i} + (x - 3y + 3z)\vec{j} + (3x + 2y - z)\vec{k}$ is solenoidal, then the value of 'k' is _____.
K:2 (a) -3 (b) 3
(c) 2 (d) 0
- CO:2 3. The value of $\iint dx dy$ over the region bounded by $x = 0$,
K:2 $x = 2, y = 0, y = 2$ is _____.
(a) 4 (b) 2
(c) 0 (d) 3
- CO:2 4. The value of $\int_0^a \int_0^a \int_0^a dx dy dz$ is _____.
K:2 (a) a^2 (b) a
(c) $3a$ (d) a^3
- CO:3 5. If V is the volume enclosed by the closed surface S then the value
K:2 of $\iint_S \vec{r} \cdot \hat{n} dS$ is _____.
(a) 3V (b) V
(c) 6V (d) V^2

CO:3 6. Work done by the force \vec{F} along the curve 'C' is given by the
K:1 integral as_____.

(a) $\int_C \vec{F} \times d\vec{r}$

(b) $\int_C \vec{F} \cdot d\phi$

(c) $\int_C \vec{F} \cdot d\vec{r}$

(d) $\int \int_C \vec{F} \cdot d\vec{r}$

CO:4 7. _____ theorem connects the relationship between line integral
K:1 and surface integral.

(a) Green's

(b) Divergence

(c) Gauss divergence

(d) Stoke's

CO:4 8. By using Stokes theorem, the value of $\int_C \vec{r} \cdot d\vec{r}$ is _____.

K:2

(a) 1

(b) 0

(c) -1

(d) 2

CO:5 9. Which one of the following function is an example for an even
K:2 function?

(a) $x + x^2$

(b) $x \cos x$

(c) $x \sin x$

(d) $x^3 \cos x$

CO:5 10. One of the Dirichlet condition for the Fourier series for the
K:1 function $f(x)$ is _____.

(a) $f(x)$ must be single-valued

(b) $f(x)$ must be multiple-valued

(c) $f(x)$ must have infinite number of continuities

(d) $f(x)$ must have infinite number of discontinuities

PART - B (5 X 5 = 25 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 250 words.

CO:1 11. (a) Obtain the directional derivative of $\phi = xy^2 + yz^3$ at the
K:3 point $(2, -1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$.

(OR)

(b) Prove that $div(r^n \vec{r}) = (n + 3)r^n$. Deduce that $r^n \vec{r}$ is solenoidal iff $n = -3$.

CO:2 12. (a) Compute the value of $I = \iint_D xy \, dx dy$ where D is the region
K:3 bounded by the curve $x = y^2, x = 2 - y, y = 0, y = 1$.

(OR)

(b) Find the area of the circle with radius r , using double integration.

CO:3 13. (a) Find the work done by the force

K:3

$$\vec{F} = 3xy\vec{i} - 5z\vec{j} + 10x\vec{k} \text{ along the curve } C,$$

$$x = t^2 + 1, y = 2t^2, z = t^3 \text{ from } t = 0 \text{ to } t = 2.$$

(OR)

(b) Estimate the value of the integral $\int_C \vec{f} \cdot d\vec{r}$, where

$\vec{f} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ and the curve C is the rectangle in the xy -plane bounded by $x = 0, y = 0, x = a, y = b$.

CO:4 14. (a) Using Green's theorem, evaluate $\int_C (xy - x^2)dx + x^2ydy$ along the closed curve C formed by $y = 0, x = 1$ and $y = x$.

K:3

(OR)

(b) Find the value of the integral $\int_C (e^x dx + 2y dy - dz)$ by using Stoke's theorem, where C is the curve $x^2 + y^2 = 4, z = 2$.

CO:5 15. (a) Express $f(x) = c$ as a Fourier sine series in the range 0 to π . Hence find the value of the series $1 - \frac{1}{3} + \frac{1}{5} + \dots$.

K:3

(OR)

(b) Expand $f(x) = x^2$ as a Fourier Series in the interval $-\pi \leq x \leq \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$.

PART - C (5 X 8 = 40 Marks)

Answer ALL Questions choosing either (a) or (b).

Answer should not exceed 500 words.

CO:1 16. (a) Prove that $\text{curl}(\text{curl } \vec{f}) = \text{grad div } \vec{f} - \nabla^2 \vec{f}$.

K:3

(OR)

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at $(4, -3, 2)$.

CO:2 17. (a) Evaluate $\iiint_D xyz dx dy dz$, where D is the region bounded by the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

K:5

(OR)

(b) Evaluate $\int_1^4 \int_{\sqrt{y}}^2 (x^2 + y^2) dx dy$, by changing the order of integration.

CO:3 18. (a) Evaluate $\iint \vec{f} \cdot \hat{n} dS$, where $\vec{f} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + 2\vec{k}$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0, x = a, y = a$ and $z = a$.

K:5

(OR)

(b) Evaluate $\iint \vec{f} \cdot \hat{n} dS$, where $\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

CO:4 19. (a) Verify Stoke's theorem for $\vec{f} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

K:4

(OR)

(b) Verify Gauss divergence theorem for the vector function $\vec{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2, z = 0$ and $z = h$.

CO:5 20. (a) Find a cosine series in the range 0 to π for

K:3

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \frac{\pi}{2} \\ \pi - x, & \text{for } \frac{\pi}{2} < x < \pi \end{cases} .$$

Hence find the value of the series $\frac{1}{1^2} + \frac{1}{3^2} + \dots$.

(OR)

(b) Express $f(x) = \frac{1}{2} (\pi - x)$ as a Fourier series with period 2π , to be valid in the interval 0 to 2π .