

# KAMARAJ COLLEGE (Autonomous)

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(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

(4 Pages)

Reg. No:.....

Question Code: 26E00202

Course Code: 24UMMA22/25UMMA22

UG Degree - End Semester Examinations, April 2026

Second Semester  
B.Sc., MATHEMATICS  
Integral Calculus

(For those who joined in July 2024 and June 2025 onwards)

Time : 3Hours

Maximum: 75 Marks

## PART - A (10 × 1 = 10 Marks)

Answer ALL Questions

Choose the correct answer :

CO:2 1. What is the value of  $\int e^{ax} dx$

- K:1
- (a)  $ae^{ax}$  (b)  $\frac{e^{ax}}{a}$   
(c)  $e^{(a+1)x}$  (d)  $\frac{e^{(a+1)x}}{a+1}$

CO:2 2. Find the value of  $\int_0^{\pi/2} \sin^7 x dx$

- K:1
- (a)  $5\pi$  (b)  $\frac{16}{35}$   
(c)  $\frac{\pi}{32}$  (d)  $\frac{16}{35}\pi$

CO:1 3. Let  $f(x)$  be a continuous function in the closed interval from  $x = a$  to  $x = b$ , then which of the following is true?

K:1

- (a)  $\int_a^b f(x)dx = F(b) - F(a)$   
(b)  $\int_a^b f(x)dx = F(b)F(a)$   
(c)  $\int_a^b f(x)dx = F(b) + F(a)$   
(d)  $\int_a^b f(x)dx = F(a) - F(b)$

C0:1 4. If the region of integration is a rectangle between the lines

K:1  $x = a, x = b, y = c, y = d$ , then  $\int_R f(x, y) dA$

(a)  $\int_a^b f(x, y) dx$

(b)  $\int_c^d \int_a^b f(x, y) dx dy$

(c)  $\int_c^b \int_a^d f(x, y) dx dy$

(d)  $\int_a^b f(x, y) dx + \int_c^d f(x, y) dx dy$

C0:3 5. Which of the following is the equation of cardioid?

K:1 (a)  $r = a(1 + \sin \theta)$  (b)  $r = a(1 + \cos \theta)$

(c)  $r = a(1 - \sin \theta)$  (d)  $r = a(1 - \cos \theta)$

C0:3 6. The centroid of the area enclosed by the parabola  $y^2 = 4ax$  is \_\_\_\_

K:1 (a)  $\left(\frac{3a}{7}, \frac{3a}{4}\right)$  (b)  $\left(\frac{3a}{2}, \frac{4a}{2}\right)$

(c)  $\left(\frac{3a}{5}, \frac{3a}{2}\right)$  (d)  $\left(\frac{3a}{5}, \frac{3a}{4}\right)$

C0:5 7. Find the value of  $\Gamma(n + 1)$ , where  $n$  is a positive integer \_\_\_\_

K:1 (a)  $(n - 1)!$  (b) 1

(c)  $(n + 1)!$  (d)  $n!$

C0:5 8. Find  $\Gamma\left(\frac{1}{2}\right)$

K:1 (a)  $\sqrt{\pi}$  (b) 1

(c)  $\sqrt{\pi} - 1$  (d) 0

C0:4 9. What is the length of the curve between two points  $(x_1, y_1)$  and  $(x_2, y_2)$ ?

K:1

(a)  $\int_{x_1}^{x_2} \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} dx$  (b)  $\int_{x_1}^{x_2} \left\{1 - \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} dx$

(c)  $\int_{x_1}^{x_2} \left\{1 + \left(\frac{dy}{dx}\right)^3\right\}^{\frac{1}{2}} dx$  (d)  $\int_{x_1}^{x_2} \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{3}} dx$

CO:4 10. What is the perimeter of the cardioid?

- K:1 (a)  $2a$  (b)  $4a$   
(c)  $3a$  (d)  $8a$

**PART - B (5 X 5 = 25 Marks)**

**Answer ALL Questions choosing either (a) or (b).**

**Answer should not exceed 250 words.**

CO:2 11. (a) Integrate  $e^{-x} \sin^2 x$

K:3 (OR)

(b) Evaluate  $\int x^3 \cos 2x dx$ .

CO:5 12. (a) By changing the order of integration, solve  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$

K:3

(OR)

(b) Solve  $\int \int xy dx dy$  taken over the positive quadrant of the circle  $x^2 + y^2 = a^2$ .

CO:4 13. (a) Find the area enclosed by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

K:4

(OR)

(b) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$ , the planes  $y + z = 4$  and  $z = 0$ .

CO:1 14. (a) Evaluate  $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$

K:4

(OR)

(b) Evaluate  $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$ .

CO:3 15. (a) Find the area bounded by one arch of the curve  $y = \sin ax$  and the  $x$  axis.

K:3

(OR)

(b) Find the area of the cardioid  $r = a(1 + \cos \theta)$

**PART - C (5 X 8 = 40 Marks)**

**Answer ALL Questions choosing either (a) or (b).**

**Answer should not exceed 500 words.**

CO:5 16. (a) If  $\int_0^{\frac{\pi}{2}} \cos^m x \cos nx dx = f(m, n)$ , prove that  $f(m, n) = \frac{m}{m+n} f(m-1, n-1)$  and also prove that  $f(n, n) = \frac{\pi}{2^{n+1}}$ .

K:5

**(OR)**

(b) Deduct the reduction formula for  $I_{m,n} = \int \sin^m x \cos^n x dx$ , where  $m, n$  are positive integers.

CO:4 17. (a) By changing into polar coordinates, evaluate the integral

K:6 
$$\int_0^{2a} \int_0^{\sqrt{(2ax-x^2)}} (x^2 + y^2) dx dy.$$

**(OR)**

(b) Change the order of integration in the  $\int_0^a \int_{x^2/a}^{2a-x} xy dx dy$  and evaluate it.

CO:2 18. (a) A plane lamina of non uniform density is in the form of a

K:6 quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . If the density at any point  $(x, y)$  be  $kxy$ , where  $k$  is a constant, find the coordinates of the centroid of the lamina.

**(OR)**

(b) Solve the centroid of a loop of the lemniscate  $r^2 = a^2 \cos 2\theta$ .

CO:1 19. (a) Evaluate in terms of gamma functions the integral

K:5 
$$\iiint x^p y^q z^r dx dy dz$$
 taken over the volume of the tetrahedron given by  $x \geq 0, y \geq 0, z \geq 0$  and  $x + y + z \leq 1$ .

**(OR)**

(b) Express  $\int_0^1 x^m (1 - x^n)^p dx$  in terms of gamma functions and evaluate the integral  $\int_0^1 x^5 (1 - x^3)^{10} dx$ .

CO:3 20. (a) Find the area of a belt of a sphere of radius  $a$  intercepted

K:3 between two parallel planes at distances  $b$  and  $c$  from the centre on the same side.

**(OR)**

(b) Find the length of one loop of the curve  $3ay^2 = x(x - a)^2$ .