

(8 pages)

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M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2024

First Semester

Physics — Core

MATHEMATICAL PHYSICS

(For those who joined in July 2023 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL the questions.

Choose the correct answer :

1. Two vectors and their resultant have the same magnitude. The angle between the vectors must be
- (a) 0°
 - (b) 60°
 - (c) 120°
 - (d) Any obtuse angle

2. The vectors $(1,0,1)$, $(0,1,1)$, $(1,1,0)$ are
- (a) Orthogonal
 - (b) Linearly independent
 - (c) Linearly dependent
 - (d) Zero
3. The triple product of $A \cdot (A \times B)$ is _____
- (a) 0
 - (b) 1
 - (c) AB
 - (d) A^2B
4. The function $f(z) = z^3$ with $z = x + iy$ is analytic
- (a) in entire z -plane
 - (b) for positive values of x and y only
 - (c) at $x = 0$, $y = 0$ only
 - (d) on the line $y = 1$ only
5. The polar form of Cauchy-Riemann equations is
- (a) $-\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
 - (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
 - (c) $\frac{\partial u}{\partial r} = -\frac{\partial v}{\partial r}, \frac{\partial u}{\partial \theta} = \frac{1}{r} \frac{\partial v}{\partial \theta}$
 - (d) $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, -\frac{\partial u}{\partial \theta} = \frac{1}{r} \frac{\partial v}{\partial r}$

6. The measure of central item is called as

- (a) Mean
- (b) Mode
- (c) Standard deviation
- (d) Median

7. Every square matrix can be uniquely expressed as

- (a) A sum of two symmetric matrices
- (b) A sum of two anti-symmetric matrices
- (c) A symmetric and an anti-symmetric matrix
- (d) Linear combination of any two matrices

8. The eigen values of an anti symmetric matrix are

- (a) ± 1
- (b) 0 or real
- (c) $\pm i$
- (d) 0 or imaginary

9. The inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is

- (a) $\frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
- (c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
- (d) Does not exist

10. The Fourier sine transform of function $f(x) = e^{-ax}$ is

- (a) $\frac{s}{a^2 - s^2}$
- (b) $\frac{a}{a^2 - s^2}$
- (c) $\frac{s \sin ax}{\sqrt{a^2 - s^2}}$
- (d) $\frac{a \sin ax}{\sqrt{a^2 - s^2}}$

11. Fourier transform of function $f(t) = e^{-t+1}$ is

- (a) $\frac{2}{s^2 + 1}$
- (b) $\frac{2}{(1+s)^3}$
- (c) $\frac{2}{1+s^2}$
- (d) $\frac{2}{s^2 + 2s + 1}$

12. Laplace transform of $\sin at$ is

- (a) $\frac{a}{s^2 - a^2}$
- (b) $\frac{s}{s^2 - a^2}$
- (c) $\frac{s}{s^2 + a^2}$
- (d) $\frac{a}{s^2 + a^2}$

13. The Hermite polynomial $H_1(n)$ is

- (a) 0
- (b) 1
- (c) x
- (d) $2x$

14. The orthogonality condition for lagendre polynomial $\int P_m(x) P_n(x) dx = 0$ is true for

- (a) $m = n$
- (b) $m \neq n$
- (c) $m = n = 0$ only
- (d) $m = 1$

15. The value of $P_3(x)$ is

- (a) $3x$ (b) $\frac{1}{2}(3x^2 - 1)$
 (c) $\frac{1}{2}(5x^3 - 3x)$ (d) $\frac{1}{2}(3x^3 - 1)$

PART B — (5 × 4 = 20 marks)

Answer ALL the questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

16. (a) Find whether the set of vectors $x_1 = (1, 2, 1)$, $x_2 = (3, 1, 5)$ and $x_3 = (3, -4, 7)$ is linearly independent or dependent.

Or

- (b) Define scalar product of two vectors. Give their characteristics.

17. (a) Derive necessary and sufficient conditions for a function to be analytic.

Or

- (b) Evaluate the following integral using Cauchy integral formula $\int \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $|z| = \frac{3}{2}$.

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18. (a) Find the rank of A where matrix

$$[A] = \begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{pmatrix}$$

Or

- (b) Find the eigen values of the following matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

19. (a) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$.

Or

- (b) Find the inverse Laplace transform of $\frac{s+1}{s^2-6s+25}$.

20. (a) Deduce Rodrigue's formula for Hermite polynomials.

Or

- (b) Derive an expression for the generating function for the Legendre polynomials.

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PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

21. (a) Describe the Gram-Schmidt orthogonalization procedure of constructing an orthogonal set of functions from a non-orthogonal.

Or

- (b) (i) Define ket and bra notation and discuss their significance.

- (ii) Define projection operator.

22. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ if $a > |b|$.

Or

- (b) Find the mode from the following data:

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

23. (a) Find the characteristic equation of the following matrix and verify the Cayley -

Hamilton theorem. $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}$.

Or

- (b) Find the diagonalization matrix A is given by

$$A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}.$$

24. (a) (i) Prove Convolution theorem in Fourier transform. (4)

- (ii) Find the Laplace transform of $\int_0^t \cosh 4t \cdot \sin 6t dt$. (4)

Or

- (b) Find Fourier sine transform and cosine transform of the function $f(x) = x^2$; $0 < x < 4$.

25. (a) Derive an expression for the generating function for the Hermite polynomials.

Or

- (b) Show that Rodrigue formula for Legendre polynomials is given by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$