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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL, 2025.

First Semester

Physics – Core

MATHEMATICAL PHYSICS

(For those who joined in July 2023 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. The vectors (1, 0, 1), (0, 1, 1), (1, 1, 0) are
- (a) Orthogonal
 - (b) Linearly dependent
 - (c) Linearly independent
 - (d) None of the above

2. A vector space with no finite basis is said to be
- (a) infinite dimensional
 - (b) n dimensional
 - (c) both (a) and (b)
 - (d) none
3. An orthogonal transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is an
- (a) Isomorphism
 - (b) Automorphism
 - (c) Homomorphism
 - (d) Epimorphism
4. If $U = x^2 - y^2$, then the corresponding analytic function is
- (a) $z^2 + c$
 - (b) $z + c$
 - (c) $z^3 + c$
 - (d) $z^4 + c$
5. The probability of getting at least two heads when tossing a coin three times is
- (a) 1/8
 - (b) 3/8
 - (c) 1/2
 - (d) 5/8
6. _____ was developed for a function which is analytic at every point inside a circle C_0 .
- (a) Taylor's series
 - (b) Laurent series
 - (c) Residue theorem
 - (d) Cauchy integral theorem

7. Which one of the following matrices is Hermitian?

- (a) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

8. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ is

- (a) 0 (b) 1
 (c) 2 (d) 3

9. The eigen values of the matrix

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \text{ is}$$

- (a) $e^{+i\theta}$ (b) $e^{+2i\theta}$
 (c) $e^{+3i\theta}$ (d) $e^{+i\frac{\theta}{2}}$

10. The Dirac delta function is called

- (a) impulse function (b) analytic function
 (c) harmonic function (d) eigen function

11. The Laplace transform of $L(e^{at})$ is

- (a) $1/s-a$ (b) 1
 (c) $1/s$ (d) $s/s^2 + a^2$

12. The convolution theorem involving Fourier theorem is

- (a) $F\{I^{(v)}\} = F\{k(x)\} \cdot F\{f(x)\}$
 (b) $F\{I(x)\} = F\{k(x)\}$
 (c) $F\{I(x)\} = F\{f(x)\}$
 (d) None

13. The integral $\int_0^\pi P_n(\cos \theta) \sin 2\theta d\theta, n > 1$ where $P_n(x)$ is the Legendre polynomial of degree n equal to

- (a) 1 (b) $1/2$
 (c) 0 (d) 2

14. The value of Hermite polynomial $H_1(n)$ is

- (a) 0 (b) 1
 (c) x (d) $2x$

15. Sturm Liouville operator is

- (a) $\frac{d}{dx} \left\{ P(x) \frac{d}{dx} \right\} + q(x)$
 (b) $\frac{d}{dx} \left\{ P(x) \frac{dy}{dx} \right\} + q(x)y$
 (c) $\frac{d}{dx} \left\{ P(x) \frac{d}{dx} + q(x) \right\}$
 (d) $\frac{d}{dx} \left\{ P(x) \frac{d}{dx} - q(x) \right\}$

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

16. (a) Define scalar product. Show that the series

$$\sum_{n=1}^{\infty} C_n^* C_n \text{ converges.}$$

Or

- (b) Explain Gram-Schmidt orthogonalization procedure.

17. (a) State and prove Cauchy's integral theorem.

Or

- (b) If the probability of a horse A winning a race is $1/5$ and the probability of a horse B winning the same race is $1/6$. What is the probability that one of the horse wins?

18. (a) Find the inverse of the given matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ 2 & 5 & 2 \end{bmatrix}$$

Or

- (b) Find the characteristic equation of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

19. (a) State and prove convolution theorem in Fourier transform.

Or

- (b) Find the Laplace transform of the function $F(t) = t^n$.

20. (a) Find the value of $\int_{-\infty}^{\infty} e^{-x^2} [H_2(x)]^2 dx$.

Or

- (b) Prove that $\int_{-1}^1 P_n(x) (1 - 2xt + t^2)^{-1/2} dx = \frac{2t^n}{2n+1}$

where n is a positive integer.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 600 words.

21. (a) Show that the following two vectors are orthogonal to each other.

$$A = 4\alpha_1 - 2\alpha_2 - \alpha_3 + 2\alpha_4 + \alpha_5 \text{ and}$$

$$B = 2\alpha_1 + 2\alpha_2 - 3\alpha_4 + 2\alpha_5 \text{ where } \alpha's \text{ form a orthogonal set.}$$

Or

- (b) Define linear vector space. Write any three examples of linear vector spaces.

22. (a) Show that $e^i(\cos y + i \sin y)$ is an analytic function. Find its derivatives.

Or

- (b) Evaluate the values of mean and standard deviation for the following data :

Size of item : 6 7 8 9 10 11 12

Frequency : 3 6 9 13 8 5 4

23. (a) Find the inverse of the matrix

$$M = \begin{bmatrix} 5 + i & 4 + 2i \\ 10 + 3i & 8 + 6i \end{bmatrix}.$$

Or

- (b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

24. (a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

Or

- (b) Find $L^{-1} \left\{ \frac{S}{(S^2 + a^2)^2} \right\}$.

25. (a) Derive Rodrigue formula for Legendre polynomial.

Or

- (b) Derive Strum Liouville's equation.

22. (a) Show that $e^x(\cos y + i \sin y)$ is an analytic function. Find its derivatives.

Or

- (b) Evaluate the values of mean and standard deviation for the following data :

Size of item : 6 7 8 9 10 11 12

Frequency : 3 6 9 13 8 5 4

23. (a) Find the inverse of the matrix

$$M = \begin{bmatrix} 5 + i & 4 + 2i \\ 10 + 3i & 8 + 6i \end{bmatrix}$$

Or

- (b) Find the eigen values and eigen vectors of the

$$\text{matrix } A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

24. (a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

Or

$$(b) \text{ Find } L^{-1} \left\{ \frac{S}{(S^2 + a^2)^2} \right\}$$

25. (a) Derive Rodrigue formula for Legendre polynomial.

Or

- (b) Derive Strum Liouville's equation.